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The experimental status of Higgs Sector of the Standard Electroweak Model at the end of the LEP-SLC era

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Abstract. A method is proposed to calculate the confidence level for agreement of data with the Higgs sector of the Standard Model (SM). This is done by combining information from direct and indirect Higgs Boson searches. Good agreement with the SM is found for $m_H \simeq 120$ GeV using the observables most sensitive to m_H: A_l and m_W. In particular, quantum corrections, as predicted by the SM, are observed with a statistical significance of forty-four standard deviations. However, apparent deviations from the SM of 3.7σ and 2.8σ are found for the $Z\nu\overline{\nu}$ and right-handed $Zb\overline{b}$ couplings respectively. The maximum confidence level for agreement with the SM of the entire data set considered is $\simeq 0.006$ for $m_H \simeq 180$ GeV. The reason why confidence levels about an order of magnitude higher than this have been claimed for global fits to similar data sets is explained.

1 Introduction

It is now almost three decades since the the first accelerator [1,2] and physicss [1,3] studies, that eventually lead to the construction and operation of the LEP e^+e^- collider at CERN were performed. Now, some roughly twenty man-millenia of work by physicists and engineers later, the almost final results of the LEP and concurrent SLC (Stanford Linear Collider) experimental programs are available [4,5]. By general consensus, the most scientifically important of these results concern high precision tests of the Standard Electroweak Model (SM) [6–8], in particular the Higgs Sector that tests the proposed mechanism of spontaneous symmetry breaking. During the same period, important contributions to this subject (discovery of, and measurement of the mass, m_t, of the top quark, measurement of the mass, m_W, of the W boson and the NuTeV neutrino-quark scattering results) were also made at the FERMILAB laboratory.

The LEP program consisted of two stages. In the first, 'Z-pole', running a total of $\simeq 1.7 \times 10^7$ Z decays into pairs of all the fundamental fermions (the matter fields of the SM), except the top quark, were collected by the four LEP detectors, ALEPH, DELPHI, L3 and OPAL. At SLC, the same processes were studied with lower statistics but with precisely controlled electron beam polarisation, allowing experiments of very high sensitivity to be performed, so producing results of comparable statistical accuracy to those of LEP. The final outcome of all this work may be summarised, in what concerns the SM, in only seven numbers, which, for definiteness can be taken to be the right- and left-handed couplings to the Z of the charged lepton, c-quark and b-quark pairs and the (left-handed) coupling of the Z to neutrino pairs. The busy reader, who would like to go straight to the final conclusions, is invited to look directly at Tables 19 and 20 below where the measured values of these coupling constants, together with the corresponding SM predictions are shown. The second, 'high energy' phase of LEP operation at collision energies above the threshold for W pair production provided essentially one additional high precision SM parameter, m_W. This measurement, in combination with the FERMILAB measurement of comparable accuracy of the same quantity, is also shown in Tables 19 and 20. Other important measurements performed during the second LEP phase were of the triple boson WWγ and WWZ couplings, but these have less impact than the fermion couplings as a test of the core physics underlying the SM: a renormalisable quantum field theory incorporating local gauge invariance that is spontaneously broken by the Higgs mechanism[1]. They are therefore not discussed further in this paper. In the conventionally used on-shell renormalisation scheme [9]

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[1] In fact the now experimentally well verified [4] SM predictions for the WWγ and WWZ couplings were aleady contained in Glashow's original electroweak paper [6] where they follow at tree level from global $SU(2)_L$ invariance and quantum mechanical mixing of the W^0 and B^0 fields. Their values then shed little light on the correctness (or otherwise) of either the renormalisabilty of the theory or the Higgs mechanism. A genuine test of local gauge symmetry would be provided by measurements of the strength of quadrilinear boson couplings. So far this has not been done.

where m_W is traded as an input parameter for the much more precisely known Fermi constant, G_μ, derived from the measured muon lifetime, the predictions of the SM depend (apart from other and better known parameters) on three relatively poorly known ones: m_t, the electromagnetic couplant at the Z mass scale, $\alpha(m_Z)$, and the mass of the Higgs boson, m_H. Because of quantum loop corrections, the right- and left-handed couplings, $\overline{g}_l^R$ and $\overline{g}_l^L$, of the Z to charged lepton pairs, as well as m_W, are strongly sensitive to the values of m_t and m_H. Actually, it is only the ratio $\overline{g}_l^R/\overline{g}_l^L$ and m_W which are strongly sensitive to m_H, so that the measurements of these two quantites provide the most stringent limits, from quantum corrections, on the value of m_H.

In the second phase of LEP, an unsuccessful search was performed for directly produced Higgs bosons [10], resulting in the 95% confidence level (CL) lower limit $m_H > 114.4$ GeV. The principle aim of the present paper is to combine, in a transparent way, this direct limit with the indirect information derived from $\overline{g}_l^R/\overline{g}_l^L$ (or equivalently A_l, see below) and m_W, to derive combined curves of $\overline{\rm CL}$, the confidence level that the m_H-sensitive data agrees with the SM, as a function of m_H. The reader might then reasonably hope that the result of the paper would be a single curve of $\overline{\rm CL}$ versus m_H. In fact she (or he) will find eight figures where $\overline{\rm CL}$ is plotted versus m_H containing in total 18 different curves. The reason for this complication is that the data, though perfectly consistent experimentally[2], is not consistent with the SM (see Tables 19 and 20) and depending on the assumptions made (SM correct, model-independent analysis, certain data included or excluded) different results are found for the $\overline{\rm CL}$ curves. I have included a number of different possibilities to demonstrate the inconsistency of the data with SM predictions. The reader may then choose the curve for which the assumptions match best her (or his) own favourite ones.

The structure of the paper is as follows: In the next section some general remarks concerning the different functions of the science of Statistics in data analysis are made. In particular it is pointed out that the choices made until now for the χ^2 estimator in global electroweak analyses give an over-optimistic estimate of the level of agreement of the data with the SM. In Sect. 3, the heavy quark asymmetry measurements that are *prima facie* inconsistent with charged lepton asymmetries, when both are interpreted within the SM, are discussed. In particular, the internal consistency of the data and systematic error estimates are examined in some detail. The sensitivities of different electroweak observables to m_t and m_H are discussed in Sect. 4. This is the only place in the paper where fit results are shown and discussed. It is demonstrated that the sensitivity to m_H comes essentially from only the observables A_l and m_W. Section 5 describes the algorithm used for combining the direct and indirect measurements of m_H. Results for $\overline{\rm CL}$ derived from A_l and m_W, assuming the correctness of the SM, but selecting different data, are shown. Also shown in this section is the sensitivity of $\overline{\rm CL}$ to the values of the parameters m_t and $\alpha(m_Z)$. In Sect. 6 the alternative interpretations of the result of the NuTeV experiment are explained and it is pointed out that the interpretation, as required in a model-independent analysis, as a measurement of the $Z\nu\overline{\nu}$ coupling, instead of m_W, is strongly favoured by arguments of statistical consistency. In Sect. 7 a complete set of model independent observables is extracted and compared with SM predictions. Constraints are set on the coupling of non-b down-type quarks using the precisely measured observable $\Gamma_{\rm had}$. Quantum corrections are extracted for different fermion flavours and compared with SM predictions. Finally, in Sect. 7, curves of $\overline{\rm CL}$ versus m_H derived from a χ^2 estimator using all or selected subsets of the considered observables (including now m_H-insensitive ones) are shown. In Sect. 8 values of $\overline{\rm CL}$ obtained as described in Sect. 7, are compared with the confidence levels of previously published global fits to similar data. The confidence levels are seen to be very consistent when the purely statistical dilution of the hypothesis testing power of the χ^2 estimators of the global fits, as discussed in Sect. 2, is taken into account. Section 8 also contains a critical discussion of m_H limits determined from $\Delta\chi^2$ plots. Section 9 contains a summary and conclusions, including the author's personal choice of three most pertinent $\overline{\rm CL}$ versus m_H plots. The busy reader is encouraged to read this section first to get a general view of the results and conclusions returning later (if still interested) to the earlier sections for more information and supporting arguments.

When the first version of the present paper was almost complete a new experimental value of m_t, 178 ± 4.3 GeV, was announced by the CDF and D0 collaborations [11]. Since the change from the previous value of 174.3 ± 5.1 GeV has a dramatic effect on the $\overline{\rm CL}$ curves, especially for large values of m_H, all such curves shown, when the contrary is not explicitly stated, use the new value of m_t. However, at the time of writing, no global fits were yet published by the EWWG and EWPDG using the new value. Therefore, in Sect. 4 where comparisons with the results of EWWG global fits are made, the old value of m_t is used. The conclusions of Sect. 8, where the consistency of the confidence levels found in the present paper with those quoted for global fits is discussed, are unaffected by the change in the measured value of m_t.

2 Statistics: Data consistency versus hypothesis testing

In the context of the analysis of experimental data, Statistics has three quite distinct roles to play. These are:

(i) To judge whether different measurements of the same physical quantity are consistent with each other, and to derive an unbiased weighted average value of the quantity.

(ii) To test the hypothesis that an ensemble of measurements of the same or different physical quantities are consistent with some theory.

[2] That is, good agreement is found between different measurements of the same experimental observables. For details see [4]

(iii) In the case of positive answers to the questions implicit in (i) and (ii), to determine numerical values of unknown or partially known parameters of a theory from the data.

In previous and current analyses of precision electroweak data performed by the LEP and SLD electroweak working groups (EWWG) [4] and the standard model sub-group of the Particle Data Group (EWPDG) [5], only the functions (i) and (iii) above are systematically performed, with little, if any, regard for (ii). In fact tests of data consistency (comparisons of different measurements of the same physical quantity) are performed by the EWWG using the χ^2 estimator, with, in general, very satisfactory results [4]. In the global fit to all data, since the point (ii) is not addressed, all relevant data is used for parameter estimation in the global fit. In the case that all of the data is in agreement with the SM this procedure gives the best, unbiased, estimate of parameter values. However, if certain sub-sets of data do not agree with the SM, biased results may be obtained using this procedure. In particular, as will be discussed below, the fitted value of m_H obtained with the current data set is biased towards higher values by about 50 GeV by just such an effect. The EWPDG also do not investigate the level of agreement of different data sub-sets with the SM and related biases, being concerned only with the function (iii), parameter estimation on the assumption that all data is correctly described by the SM [12]. In this case different measurements of the same physical quantity are included as independent data in the fit without any prior consistency checks such as those performed by the EWWG. In the case that subsets of data do not agree with the SM, fitted prameters may then be biased in just the same way as in the EWWG global fits. It seems to the present writer that seeking the answer to the question posed in (ii) above should be the principle aim of experimental investigations of the SM, but, as a point of fact, this is an avowed goal of neither the EWWG nor the EWPDG.

So what is the answer to the question implicit in (ii) above provided by the current electroweak data set? The nature of the problem is well illustrated by some fit results quoted in a paper devoted to a search for possible evidence of supersymmetry in precision electroweak data [13]. Fitting, as a preamble, the minimal electroweak standard model to only the $\sin^2\Theta_{\rm eff}^{\rm lept}$ values derived from either leptonic or hadronic asymmetry measurements, a χ^2 per degree of freedom (χ^2/d.o.f.) of 18.4/4 was obtained corresponding to a CL of 0.001. A fit by the EWWG to the same data set, but using instead about 20 observables[3] reported a χ^2/d.o.f. of 26.0/15, with a CL of 0.04. An analysis of essentially the same year 2000 data set by EWPDG, but fitting the SM to more than 40 observables found, for a global fit, a χ^2/d.o.f. of 42/37 with a CL of 0.27 [14]. The fitted value of m_H, was very similar in these three different fits since, as discussed below, almost all the sensitivity to m_H is found in only two observables, $\sin^2\Theta_{\rm eff}^{\rm lept}$ and m_W. Thus, for essentially the same fitted value of m_H, CLs differing by a factor of up to 270, according to the fit procedure used, were obtained. Which (if any) of the different CLs most truly reflects the agreement between the data and the SM prediction? The principal aims of the present paper are, firstly, to provide an answer to this question, and, secondly, to combine CLs derived from direct and indirect experimental limits on m_H so has to obtain an meaningful overall CL that reflects both the internal consistency of different observables and the global level of agreement with the SM.

The explanation of the poor CL obtained in the fit to only the leptonic and hadronic $\sin^2\Theta_{\rm eff}^{\rm lept}$ values is now well known. As first pointed out in analyses of the 1996 data set [15,16] The Z-boson b-quark couplings appear to be anomalous at about the three standard deviation level. These couplings are quite insensitive, in the SM, to m_H and m_t, but, due to a correlation effect, when heavy quark forward/backward asymmetries are analysed, assuming the correctness of the SM, to extract a value of $\sin^2\Theta_{\rm eff}^{\rm lept}$, the latter is found to correspond to a much larger value of m_H than that derived from purely leptonic measurements [17,18]. This leads to barely compatible values of $\sin^2\Theta_{\rm eff}^{\rm lept}$ from leptonic and hadronic (essentially b-quark) data and explains the poor CL of the fit to this data to obtain m_H and m_t mentioned above.

More recently, much more precise experimental measurements of m_W have become available. These are found to favour a value of m_H almost as low as that suggested by the leptonic data, thus resulting in a large discrepancy between the m_H value obtained by combining the leptonic data and m_W and that derived from hadronic asymmetries. This problem has been has been recently stressed in the literature [19] and is now generally appreciated [20].

The reason for the factor $\simeq 300$ difference in the CLs of different fits is easily understood. The point is that the fit to only the $\sin^2\Theta_{\rm eff}^{\rm lept}$ values was essentally performing the function (ii) above, i.e. hypothesis testing, whereas the EWWG and EWPDG fits were combining the functions (i) and (ii) with a large weighting factor in favour of (i). How this happens will now be explained. In addition to this effect, the hypothesis testing ability of the fit χ^2 is further blunted by the inclusion of observables in the fit, that have almost no sensitivity to m_H and m_t, in both the EWWG and EWPDG analyses.

Consider a number, N, of independent measurements, Q_i, of the same quantity, Q. The theoretical expectation for Q is $Q_{\rm Thy}$ and the weighted everage value of the measurements is $\bar{Q}$. With the assumption of uncorrelated experimental errors, three different Pearson χ^2 estimators may be defined, as follows:

$$\chi^2_{\rm data,WA} = \sum_{i=1}^{N}\frac{(Q_i-\bar{Q})^2}{\sigma_i^2} \qquad (2.1)$$

$$\chi^2_{\rm data,Thy} = \sum_{i=1}^{N}\frac{(Q_i-Q_{\rm Thy})^2}{\sigma_i^2} \qquad (2.2)$$

[3] Many of these quantities are actually ‘pseudo-observables’, but for brevity the term ‘observable’ will be used throughout this paper for extracted physical quantities sensitive to parameters of the electroweak theory.

$$\chi^2_{\rm WA,Thy} = \frac{(\bar{Q} - Q_{\rm Thy})^2}{\bar{\sigma}^2} \tag{2.3}$$

In (2.3), $\bar{\sigma}$ is the weighted mean error on the quantity $\bar{Q}$. Assuming uncorrelated, Gaussian distributed, errors it is given by the relation:

$$\frac{1}{\bar{\sigma}^2} \equiv \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \tag{2.4}$$

where σ_i is the estimated RMS uncertainty on Q_i. Noting the identity:

$$Q_i - Q_{\rm Thy} \equiv (Q_i - \bar{Q}) + (\bar{Q} - Q_{\rm Thy}) \tag{2.5}$$

(2.2) may be written as:

$$\begin{aligned}
\chi^2_{\rm data,Thy} &= \sum_{i=1}^{N}\left[\frac{(Q_i - \bar{Q})^2}{\sigma_i^2} + \frac{(\bar{Q} - Q_{\rm Thy})^2}{\sigma_i^2} + \frac{2(Q_i - \bar{Q})(\bar{Q} - Q_{\rm Thy})}{\sigma_i^2}\right] \\
&= \chi^2_{\rm data,WA} + (\bar{Q} - Q_{\rm Thy})^2 \sum_{i=1}^{N}\frac{1}{\sigma_i^2} \\
&\quad + 2(\bar{Q} - Q_{\rm Thy})\left[\sum_{i=1}^{N}\frac{Q_i}{\sigma_i^2} - \bar{Q}\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right] \\
&= \chi^2_{\rm data,WA} + \chi^2_{\rm WA,Thy} \\
&\quad + 2(\bar{Q} - Q_{\rm Thy})\left(\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\right)\left[\frac{\sum_{i=1}^{N}\frac{Q_i}{\sigma_i^2}}{\sum_{i=1}^{N}\frac{1}{\sigma_i^2}} - \bar{Q}\right] \\
&= \chi^2_{\rm data,WA} + \chi^2_{\rm WA,Thy}
\end{aligned} \tag{2.6}$$

where, in the third line of (2.6) the definition of $\bar{\sigma}$, (2.4), and (2.3) have been used, and in the fourth line the definition of $\bar{Q}$:

$$\bar{Q} \equiv \frac{\sum_{i=1}^{N}\frac{Q_i}{\sigma_i^2}}{\sum_{i=1}^{N}\frac{1}{\sigma_i^2}} \tag{2.7}$$

So, in the simple case of uncorrelated Gaussian errors, the χ^2 for consistency of the data with the theory is equal to the simple sum of the χ^2 for consistency of the data with its weighted average plus the χ^2 for consistency of the theory with the weighted average. Clearly $\chi^2_{\rm data,WA}$ is a measure only of the internal consistency of the data and so the corresponding CL provides an answer only to the question raised in point (i) above. $\chi^2_{\rm WA,Thy}$ gives, providing the CL for $\chi^2_{\rm data,WA}$ is acceptable, an estimate of probability that the data is correctly described by the theory, and so provides the hypothesis test mentioned in (ii) above, as well as estimating the values of unknown parameters of the theory (for example, m_H if $Thy = $ SM) in accordance with point (iii) above. However, the value of $\chi^2_{\rm data,Thy}$, the statistical estimator universally used by both the EWWG and the EWPDG, reflects *both* the internal consistency of the data *and* the level of agreement of the data with the theory. If the number of data is very large, the relative contribution of $\chi^2_{\rm WA,Thy}$ to $\chi^2_{\rm data,Thy}$ becomes very small, since the former χ^2 has only one degree of freedom. Under these circumstances, the CL of $\chi^2_{\rm data,Thy}$ is not a meaningful indicator of the level of agreement of the data and the theory.

To take a simple example, suppose that there are 40 data and that $\chi^2_{\rm data,WA} = 30$ and $\chi^2_{\rm WA,Thy} = 16$, so that, on the assumption of uncorrelated Gaussian errors, (2.6) gives $\chi^2_{\rm data,Thy} = 46$. The corresponding confidence levels are: $\chi^2_{\rm data,WA}$/d.o.f. = 30/39, CL= 0.849 ; $\chi^2_{\rm WA,Thy}$/d.o.f. = 16/1, CL= 6.3×10^{-5}; $\chi^2_{\rm data,Thy}$/d.o.f. = 46/40, CL= 0.28. Thus the effect of the four standard deviation discrepancy observed in $\chi^2_{\rm WA,Thy}$ is diluted to give an innocuous CL of 0.28 for the statistical estimator $\chi^2_{\rm data,Thy}$.

To take properly into account both the internal consistency of different measurements of the same quantity, and the level of agreement of the data with theory, a useful statistical procedure is to combine the confidence levels of the appropriate χ^2 functions. Since $\chi^2_{\rm data,WA}$ and $\chi^2_{\rm WA,Thy}$ are independent statistical estimators, the corresponding CLs may be combined by use of the formula [21]:

$${\rm CL}(\alpha_1, \alpha_2) = \alpha_1\alpha_2[1 - \ln(\alpha_1\alpha_2)] \tag{2.8}$$

where α_1 and α_2 are the two independent CLs to be combined. It follows that in the simple example considered above the combined CL has the value 5.8×10^{-4} so that the data/theory discrepancy is still well in evidence. Note that the combined CL is a factor 493 smaller than the CL of $\chi^2_{\rm data,Thy}$ in this case! In the following the combined CL given by (2.8) will be used to calculate the overall confidence level that the relevant data are consistent and that the data are in agreement with the SM, for different values of m_H.

In the above example each datum has the same sensitivity to the parameters of the theory. However, among the $\simeq$ 20 observables included in the global electroweak fits performed by the EWWG and the $\simeq$ 40 in the similar EWPDG fits, the majority are only weakly sensitive to the values of m_H and m_t. This effect dilutes even further the hypothesis testing power of the the statistical estimator $\chi^2_{\rm data,Thy}$ beyond that due to the dominant contribution of $\chi^2_{\rm data,WA}$ discussed above. In the statistical analysis presented below, the separate contributions of $\chi^2_{\rm data,WA}$ and $\chi^2_{\rm WA,Thy}$ to $\chi^2_{\rm data,Thy}$ will be extracted to provide separate answers to the questions posed in points (i) and (ii) above. The overall CL will then be calculated according to (2.8) above. In the case of a small number of sensitive observables [4] it will be found that, unlike in the example discussed above, good agreement is found between the CL of the total χ^2: $\chi^2_{\rm data,WA} + \chi^2_{\rm WA,Thy}$ and the combined CL calculated using (2.8). The former CL is then used as a statistical estimator for the indirect Higgs mass analysis. It is in any case important, to avoid dilution of the hypothesis-

[4] Indeed, for m_H, there are only two such observables A_l and m_W as discussed in Sect. 4 below.

testing power of the fits, to quote the results using only data which is sensitive to the parameters of main interest, m_t and m_H. This has not been done, in any systematic manner, in EWWG and EWPDG fits.

Previous authors [22,23] have calculated normalised probability density functions (PDFs) giving the relative probability of different values of m_H, by combining direct and indirect limits. Instead, in the present paper, the combined CL is found by combining the CLs of the direct and indirect measurements in region of overlap using (2.8). This combined CL gives an absolute rather than a relative probability that the SM is consistent with the data for any value of m_H. In this way the hypothesis testing aspect of the comparison of the data with the SM is addressed. This is not done by the normalised PDFs derived in [22,23].

3 Heavy quark asymmetry measurements

A discussion of the consistency of the b-quark asymmetry measurements in the data up to 1999 may be found in [24]. The current LEP and SLD heavy flavour asymmetry measurements are collected in Table 1 (b-quarks) [25] and Table 2 (c-quarks) [25]. In Table 1 are reported eight independent measurements of the forward/backward asymmetry $A_{\mathrm{FB}}^{0,\mathrm{b}}$ as well as the direct SLD measurement of A_b from the forward/backward,left/right asymmetry. Table 2 contains seven LEP measurements of $A_{\mathrm{FB}}^{0,\mathrm{c}}$ and the direct A_c measurement from SLD. For each LEP asymmetry measurement the corresponding value of A_b or A_c is estimated using the relation:

$$A_Q = \frac{4A_{\mathrm{FB}}^{0,Q}}{3A_l} \quad (Q = b, c) \tag{3.1}$$

where A_l is the LEP+SLD average value of the charged lepton asymmetry parameter extracted by assuming charged lepton universality[5]:

$$A_l = \frac{2\overline{v}_l\overline{a}_l}{\overline{v}_l^2 + \overline{a}_l^2} = \frac{2\overline{r}_l}{1+\overline{r}_l^2} \tag{3.2}$$

where

$$\overline{r}_l \equiv \frac{\overline{v}_l}{\overline{a}_l} = 1 - 4\sin^2\Theta_{\mathrm{eff}}^{\mathrm{lept}} \tag{3.3}$$

The value used is [4][6]:

$$A_l = 0.1501(16) \tag{3.4}$$

The values of A_b and A_c derived in this manner are presented in the last columns of Tables 1 and 2. The SM predictions for the values of A_b and A_c are 0.935 and 0.668 respectively, with a negligible dependence on m_H and m_t at the scale of the present experimental errors. Also shown in Tables 1 and 2 are the LEP average values of $A_{\mathrm{FB}}^{0,\mathrm{b}}$, A_b, $A_{\mathrm{FB}}^{0,\mathrm{c}}$ and A_c as well as the LEP+SLD combined values of A_b and A_c. The uncertainties on the LEP average values of A_b and A_c come mainly from those on $A_{\mathrm{FB}}^{0,\mathrm{b}}$ and $A_{\mathrm{FB}}^{0,\mathrm{c}}$ (1.6 % and 5.0 %) rather than that on A_l (1.1 %). The statistical and systematic errors on both the LEP average and the SLD measurements of A_b and A_c are of comparable magnitude.

Values of the different χ^2 estimators: $\chi^2_{\mathrm{data,WA}}$, $\chi^2_{\mathrm{WA,Thy}}$, and $\chi^2_{\mathrm{data,Thy}}$ introduced above for the quantities $Q = A_b, A_c$ are presented in Table 3[7]. The $\chi^2_{\mathrm{data,WA}}$ CLs of 0.92 and 0.90 for A_b and A_c indicate good internal consistency of the data, but also, possibly, an over-estimate of systematic errors. The $\chi^2_{\mathrm{WA,Thy}}$ CLs of 1.11×10^{-2} and 0.45 for A_b and A_c indicate in the former case a 2.5σ dis-

Table 1. The LEP and SLD measurements of b-quark asymmetry parameters. When two uncertainties are quoted, the first is statistical, the second systematic. To extract A_b the world average value: $A_l = 0.1501(16)$ [4] is used.

Experiment	$A_{\mathrm{FB}}^{0,\mathrm{b}}$	A_b
ALEPH leptons	0.1009(38)(17)	0.896(39)
DELPHI leptons	0.1031(51)(24)	0.916(51)
L3 leptons	0.1007(60)(35)	0.895(70)
OPAL leptons	0.0983(38)(18)	0.873(38)
ALEPH inclusive	0.1015(25)(12)	0.902(27)
DELPHI inclusive	0.0984(30)(15)	0.874(32)
L3 jet-charge	0.0954(101)(56)	0.847(103)
OPAL inclusive	0.1000(34)(18)	0.888(35)
SLD	–	0.925(14)(14)
LEP average	0.0997(14)(7)	0.885(14)(10)
LEP+SLD average	–	0.902(13)

Table 2. The LEP and SLD measurements of c-quark asymmetry parameters.When two uncertainties are quoted, the first is statistical, the second systematic. To extract A_c the world average value: $A_l = 0.1501(16)$ [4] is used

Experiment	$A_{\mathrm{FB}}^{0,\mathrm{c}}$	A_c
ALEPH leptons	0.0733(53)(36)	0.651(57)
DELPHI leptons	0.0724(84)(62)	0.643(93)
L3 leptons	0.0832(301)(197)	0.739(320)
OPAL leptons	0.0642(51)(37)	0.570(56)
ALEPH D*	0.0696(85)(33)	0.618(81)
DELPHI D*	0.0693(87)(27)	0.615(81)
OPAL D*	0.0759(109)(57)	0.674(109)
SLD	–	0.670(20)(16)
LEP average	0.0706(31)(17)	0.627(26)(19)
LEP+SLD average	–	0.653(20)

[5] The notation follows that of [17]

[6] Errors are quoted on the least significant digits. e.g. 4.123(32) means 4.123 ± 0.032. When two errors are quoted, the first is statistical and the second systematic

[7] In calculating the χ^2 values all errors are assumed to be uncorrelated. The common systematic uncertainity of the $A_{\mathrm{FB}}^{0,\mathrm{b}}$ measurements is only 0.0004 [25] which may be neglected as compared the the statistical and uncorrelated systematic uncertainties on $A_{\mathrm{FB}}^{0,\mathrm{b}}$.

Table 3. Different χ^2 estimators and CLs derived from the LEP and SLD measurements of A_b and A_c

	$\chi^2_{\mathrm{data,WA}}$/d.o.f., CL	$\chi^2_{\mathrm{WA,Thy}}$/d.o.f., CL	$\chi^2_{\mathrm{data,Thy}}$/d.o.f., CL	Comb. CL
A_b	3.2/8, 0.92	6.4/1, 0.011	12.0/9, 0.21	0.057
A_c	3.2/7, 0.90	0.56/1, 0.45	4.1/8, 0.85	0.77

crepancy, and, in the latter, good agreement with the SM prediction. As in the example discussed above, the A_b discrepancy is not evident in the CL of $\chi^2_{\mathrm{data,Thy}}$, which takes the value 0.21. The combined CLs, according to (2.8), that the A_b and A_c data are both consistent and in agreement with the SM are 0.057 and 0.77 repectively. This would seem to indicate that it is not unreasonable that the deviation of A_b from the SM prediction could be due to statistical fluctuation perhaps in combination with some unknown systematic effect. However, this conclusion requires confidence in the estimation of the systematic errors. As will be discussed below, there is some evidence, from the data itself, that the uncorrelated systematic errors may be somewhat overestimated, thus reducing from its true value the significance of the observed A_b deviation.

It is interesting to note that a goodness-of-fit estimator, independant of the χ^2 test, is provided by the so-called 'Run Test' [26]. For the case of the A_b data in Table 1, since all 9 independent measurements constitute a single 'run' (they are all less than the SM prediction) the corresponding CL is easily calculated. Since a single run can occur in only two ways (all data higher than or all data lower than the theoretical prediction) the CL is $2/2^9 = 3.9 \times 10^{-3}$. Unlike for the χ^2 test, the CL of the Run Test is insensitive to over- or under-estimation of uncorrelated systematic errors. Since the CLs of the Run Test, of $\chi^2_{\mathrm{data,WA}}$ and of $\chi^2_{\mathrm{WA,Thy}}$ are all independent, they may be combined into a single CL using the formula that generalises (2.8) to the case of three independent CLs: α_1, α_2 and α_3 [21]:

$$\mathrm{CL}(\alpha_1,\alpha_2,\alpha_3) = \alpha_1\alpha_2\alpha_3[1-\ln(\alpha_1\alpha_2\alpha_3) + \frac{[\ln(\alpha_1\alpha_2\alpha_3)]^2}{2}] \quad (3.5)$$

The combined CL for the A_b data given by (3.5) is 2.5×10^{-3}. The single run of the A_b data may be associated with a genuine deviation of the data from the SM prediction or a large correlated systematic effect of unknown origin. It is argued below that the latter explanation is unlikely. The third possible explanation, a statistical fluctuation, is also unlikely, given the small value of the combined confidence level.

Because of the symmetry between A_l and A_Q in (3.1), the $A^{0,\mathrm{b}}_{\mathrm{FB}}$ measurements in Table 1 can be converted to A_l values, denoted as $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}})$, using the directly measured SLD value $A_b(\mathrm{SLD}) = 0.925(20)$. This gives a LEP average result: $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}}) = 0.1437(39)$ to be compared with the LEP+SLD average value: $A_l(\mathrm{LEP}+\mathrm{SLD}) = 0.1501(16)$. All eight values of A_l derived from the measurements of $A^{0,\mathrm{b}}_{\mathrm{FB}}$ in Table 1 are less than this value. The corresponding 'run test' CL is $2/2^8 = 7.8 \times 10^{-3}$. Combining this with the CL for mutual consistency of $A_l(\mathrm{LEP}+\mathrm{SLD})$ and $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}})$ of 0.13 using (2.8) gives an overall CL of 7.9×10^{-3} comparable with, but somewhat larger than, the value obtained by comparing the different A_b values in Table 1 with the SM prediction. Again, a purely statistical fluctuation of this size is unlikely. Notice, however, that the hypothesis that the anomalous value of $A^{0,\mathrm{b}}_{\mathrm{FB}}$ is due to A_l rather than A_b is strongly disfavoured statistically. If the true value of A_l is identified with $A_l(\mathrm{LEP}+\mathrm{SLD})$ (i.e it is assumed that the $A^{0,\mathrm{b}}_{\mathrm{FB}}$ anomaly is due to A_b) it differs from the derived value of $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}})$ by only 1.64σ (CL = 0.10). On the other hand, if the true value of A_l is associated with the value of $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}})$ it differs from $A_l(\mathrm{LEP}+\mathrm{SLD})$ by 4.0σ (CL $= 6.3 \times 10^{-5}$). Thus the hypothesis that the $A^{0,\mathrm{b}}_{\mathrm{FB}}$ anomaly is entirely associated with A_l is $\simeq 1600$ times less likely than that it is entirely associated with A_b. This is a consequence of the small uncertainty on $A_l(\mathrm{LEP}+\mathrm{SLD})$ as compared to that on $A_l(A^{0,\mathrm{b}}_{\mathrm{FB}})$. Of course, intermediate hypotheses where the anomaly is associated partially with A_b and partially with A_l cannot be excluded.

As there are $\simeq 10$ independent measurements of both A_b and A_c, it is possible to compare errors estimated directly from the data, with the calculated statistical and estimated uncorrelated systematic errors on the weighted average values of A_b and A_c shown in Tables 1 and 2 . The estimators for the error on the weighted average, $\bar{\sigma}$ and its RMS uncertainty $\sigma_{\bar{\sigma}}$ are given by the formulae [27]:

$$\bar{\sigma} = \sqrt{\frac{\sum_i (Q_i - \bar{Q})^2}{N(N-1)}} \quad (3.6)$$

$$\sigma_{\bar{\sigma}} = \frac{\bar{\sigma}}{\sqrt{2N(N-1)}} \quad (3.7)$$

These formulae yield values of $\bar{\sigma}$ of 0.0089(22) for A_b, and 0.018(5) for A_c, to be compared with the estimated uncorrelated [8] errors on the WA values of 0.011 and 0.020 respectively in these quantities. The agreement is good for A_c, but for A_b it cannot be excluded that the uncorrelated systematic errors may be slightly overestimated. This is confirmed by calculation of the WA statistical error on the LEP+SLD weighted average value of A_b, which

[8] Estimating the uncorrelated and correlated contributions to the uncertainties in the LEP+SLD weighted averages yield the results: $A_b = 0.902(11)(6)$, $A_c = 0.653(20)(5)$ where the first (second) uncertainties are uncorrelated (correlated). For A_b the correlated error in mainly associated with A_l, whereas for A_c, $A^{0,\mathrm{c}}_{\mathrm{FB}}$ and A_l give roughly equal contributions.

Table 4. Measured values of A_f and $\overline{s}_f$ compared to SM predictions for $m_t = 174$ GeV, $m_H = 100$ GeV. Dev(σ) = (Meas.-SM)/Error

	leptons		c quarks		b quarks	
	A_l	$\overline{s}_l$	A_c	$\overline{s}_c$	A_b	$\overline{s}_b$
Meas.	0.1501(16)	0.25268(26)	0.653(20)	0.2897(50)	0.902(13)	0.3663(13)
SM	0.1467	0.25272	0.6677	0.2882	0.9347	0.3647
Dev(σ)	2.1	−0.15	−0.74	0.3	−2.5	1.2

is just 0.0088, in perfect agreement with the value of $\bar{\sigma}$ estimated directly from the data, and consistent with the absence of any systematic error. Using only this statistical error to calculate $\chi^2_{\mathrm{WA,Thy}}$ gives χ^2/d.o.f. = 14.0/1, CL = 1.8×10^{-4} a 3.7σ effect. Thus, although (see Table 1) the estimated systematic error on the LEP+SLD average value of A_b is by no means the dominant one, the significance of the apparent deviation of A_b from the SM prediction is very sensitive to it.

The dominant source of correlated systematic error on both $A_{\mathrm{FB}}^{0,\mathrm{b}}$ and $A_{\mathrm{FB}}^{0,\mathrm{c}}$ arises from the QCD corrections [28] of $(2.96 \pm 0.40)\%$ for $A_{\mathrm{FB}}^{0,\mathrm{b}}$ and $(3.57 \pm 0.76)\%$ for $A_{\mathrm{FB}}^{0,\mathrm{c}}$. This source contributes 35% of the total systematic error on the LEP average value of A_b, the remaining part being essentially uncorrelated between the different measurements. Thus the observed fractional discrepancy, 0.053, between the LEP average value of A_b and the SM prediction, is about 1.8 times larger than the QCD correction and about 13 times larger than the estimated uncertainty on this correction. The latter would have to have been underestimated by more than an order of magnitude in order to explain the observed discrepancy with the SM prediction. This seems unlikely. Note, however, that the estimate of systematic error from the data itself using (3.6) and (3.7) gives no information on such a correlated uncertainty.

In conclusion, the different measurements of A_b are in very good agreement with each other, but their average value shows a -2.5σ deviation from the SM prediction. The correlated systematic error must have been underestimated by a large factor if the origin of the A_b deviation is unknown systematics rather than a breakdown of the SM. The measurements of A_c, on the other hand are found to be both consistent and in good agreement (within their much larger errors) with the SM prediction. Also however, as will be seen below, all of the hadronic asymmetries show similar fractional deviations from the SM parameters favoured by the purely leptonic data. The possiblity of deviations from the SM in the c-quark and light quark sectors as large as that observed in the b-quark sector is therefore not excluded by the asymmetry data.

4 Sensitivities of electroweak observables to m_t and m_H

To justify the restricted choice of observables used below to calculate the χ^2 estimators for the data/SM comparison this section presents some results of fits to obtain m_H, or m_H and m_t. The overall approach used is the 'model-independent' one of [16–18] All charged lepton and heavy quark measurements from LEP and SLD are combined to obtain the independent observables: A_l, $\overline{s}_l$, A_b, $\overline{s}_\mathrm{b}$, A_c and $\overline{s}_\mathrm{c}$. The A_f $(f = l, b, c)$ parameters are defined as in (3.2) above, with small additional correction terms in the case of A_b. The quantity, $\overline{s}_f$, is defined as $\overline{s}_f = \overline{v}_f^2 + \overline{a}_f^2$ and so is proportional to the partial width for $\mathrm{Z} \rightarrow \mathrm{f}\overline{\mathrm{f}}$ decays. Again, due to the large mass of the b-quark, small corrections are included in this case [17]. The LEP+SLD average values of these observables used in the fits presented below are shown in Table 4. To take properly into account error correlations, the directly measured values; $A_b = 0.925(20)$ and $A_c = 0.670(26)$ from SLD are assigned speparate terms in the χ^2 estimator. Correlations between A_l, A_b and A_c resulting from (3.1) are included in the χ^2 error matrix. Also shown in Table 4 are the SM predictions for $m_t = 174$ GeV and $m_H = 100$ GeV as well as normalised deviations. This Table has the same format and SM predictions as Table 3 of [17], with which it may be directly compared.

The sensitivity of different observables to m_t and m_H is presented in Table 5 [9]. To take into account both the intrinsic sensitivity and the effect of experimental uncertainty, the quantities $(\Delta X/\sigma_X)_{m_t}$ and $(\Delta X/\sigma_X)_{m_H}$ are shown for each observable, X, with experimental uncertainty σ_X. The quantity ΔX in $(\Delta X/\sigma_X)_{m_t}$ is the change in the value of X for a variation of m_t from 164 GeV to 184 GeV, with $m_H = 120$ GeV and ΔX in $(\Delta X/\sigma_X)_{m_H}$

Table 5. Sensitivities of different measured quantites to m_t and m_H (see text)

X	X_{expt}	σ_X	$(\Delta X/\sigma_X)_{m_t}$	$(\Delta X/\sigma_X)_{m_H}$
A_l	0.1501	0.0016	3.1	−1.74
$\overline{s}_l$	0.25268	0.00026	2.4	−0.70
A_c	0.653	0.020	0.10	−0.045
$\overline{s}_\mathrm{c}$	0.2897	0.0050	0.18	−0.067
A_b	0.902	0.013	0.012	−0.017
$\overline{s}_\mathrm{b}$	0.3663	0.0013	−0.12	−0.27
$\overline{s}_\nu$	0.5014	0.0015	0.77	−0.16
$\overline{s}'_{\mathrm{nb}}$	1.3211	0.0043	0.93	−0.32
m_W	80.426	0.034	3.5	−1.2

[9] Note that the $(\Delta X/\sigma_X)_{m_t}$ entries of $\overline{s}_l$ and $\overline{s}_\mathrm{b}$ of the similar table in [17] are incorrect.

Table 6. Different experimental determinations, derived assuming the correctness of the SM, of the leptonic asymmetry parameter A_l

Source	leptons	b-quarks	c-quarks	$Q_{\rm FB}^{\rm had}$	hadronic mean	overall mean
A_l	0.1501(16)	0.1422(23)	0.1423(72)	0.1401(95)	0.1421(21)	0.1472(13)

is the change in the value of X for a variation of m_H from 100 GeV to 200 GeV, with $m_t = 174.3$ GeV. Most of the sensitivity to m_t resides in the observables A_l, $\overline{s}_l$ and m_W, to m_H in A_l and m_W, and, to a lesser extent, in $\overline{s}_l$. The greater sensitivity of $\overline{s}_l$ (or Γ_l) to m_t than to m_H has also been noted in a recent paper [29].

Also included in Table 5 are the observables: $\overline{s}_\nu = \overline{v}_\nu^2 + \overline{a}_\nu^2$ and $\overline{s}'_{\rm nb}$ (to be discussed below) which is similarly defined to $\overline{s}_{\rm c}$ and $\overline{s}_{\rm b}$ but for non-b quarks. Both these observables have a moderate sensitivity to m_t and a much smaller one to m_H.

As pointed out above, if $A_{\rm FB}^{0,\rm b}$ is used as observable to estimate, via quantum corrections, m_H, a very different value is obtained from that favoured by A_l or m_W. This is due to the linear dependence of $A_{\rm FB}^{0,\rm b}$ on A_l (see (3.1) above) and the 2.5σ deviation of A_b from the SM prediction discussed in the previous section[10]. Assuming the correctness of the SM, 'hadronic' values of A_l may be extracted from the measurements of $A_{\rm FB}^{0,\rm b}$ and $A_{\rm FB}^{0,\rm c}$ by substituting the SM predictions for A_b ,A_c (which are essentially independent of m_H and m_t) into (3.1). Another, independent 'hadronic' value of A_l may be derived from the value of $\sin^2\Theta_{\rm eff}^{\rm lept}$ obtained from the SM analysis the quark anti-quark charge asymmetry, $Q_{\rm FB}^{\rm had}$ [4]. These different 'hadronic' determinations of A_l, obtained by assuming the correctness of the SM, are presented, together with the 'leptonic' value from Table 4, in Table 6. Note that the 'leptonic' value of A_l, although derived assuming charged lepton universality does *not* assume the correctness of the SM, only that the process $\rm Z \rightarrow l\overline{l}$ is described by some effective vector and axial vector couplings so that (3.1) is obeyed.

The 'leptonic' value of A_l in Table 6 is derived from e, μ and τ forward/backward charge asymmetries and from τ-polarisation measurements. The hadronic ones from quark forward/backward charge asymmetries. In fact the A_l derived uniquely from τ-polarisation measurements: $A_l(\tau - poln) = 0.1465(33)$ lies almost exactly mid-way between the SLD ALR and LEP $A_{\rm FB}^{0,l}$ weighted average of 0.1513(19) and the value of A_l(had) quoted in Table 6. It is 1.3σ below the former and 1.1σ above the latter. In the 1996 data set [30] the difference between $A_l(\tau - poln)$ and the ALR, $A_{\rm FB}^{0,l}$ average was much larger, 2.5σ, so that the inclusion (or not) of the τ-polarisation data had a large effect on the value of A_l extracted using (3.1). This was discussed in some detail in [16]. In another paper discussing the same 1996 data set [31] it was pointed out that, considering also the τ-polarisation data as 'hadronic' (because of the predominantly hadronic final states), the value of $\sin^2\Theta_{\rm eff}^{\rm lept}$ derived from the leptonic ALR and $A_{\rm FB}^{0,l}$ measurements was found to differ by more than 3σ from that given by the 'hadronic' ones, i.e. τ-polarisation and quark asymmetry measurements. The situation is much improved in the current (essentially final) LEP+SLC data. Excluding the τ-polarisation measurements gives a minor change in the WA A_b value: $A_b(\tau - poln\ out) = 0.898(13)$ to be compared with the value quoted in the last row of Table 1. The deviation of A_b from the SM prediction is only increased from 2.5σ to 2.8σ, instead of the $\simeq 1\sigma$ increase found in the 1996 data set [16].

Because of its strong dependence on m_H and m_t then, unlike in the case of A_b and A_c, no definite SM prediction exists for A_l. However it is of interest to compare the 'leptonic' value of A_l, A_l(lept) with the different 'hadronic' values A_l(had). The following χ^2 values and confidence levels are obtained: $\chi^2_{\rm had,WA}$/d.o.f. = 0.047/2, CL= 0.977; $\chi^2_{\rm all,WA}$/d.o.f. = 9.0/3, CL= 0.029: $\chi^2_{\rm had,lept}$/d.o.f. = 9.2/1, CL= 0.0024. Thus the three hadronic determinations are very consistent with each other, whereas the hadronic and leptonic determinations differ by 3 standard deviations. This poor overall consistency of the different values of A_l, extracted assuming the correctness of the SM for the quark couplings, must be taken into account when assessing the overall level of agreement of the data with the SM.[11] It is important to stress that this mismatch is not the result of any inconsistency evident in the experimental data themselves, but rather the result of interpreting the data according to the SM prediction.

The following strategy is now followed for fits to obtain limits on m_H: In a first step, fits similar to those previously presented in [17,18] are performed to the the entire LEP+SLD data set contributing to the six observables of Table 4, as well as LEP+FERMILAB combined direct measurement of m_W: $m_W = 80.426(34)$ GeV. Other fits are done including also the indirect determination of m_W: m_W(NuTeV) = 80.136(83) GeV by NuTeV [32]. Only m_H is varied in the fits, the other important parameters: m_Z, m_t, $\alpha(m_Z)$ and $\alpha_s(m_Z)$ being fixed at their measured values [12]of 91.1875 GeV, 174.3 GeV, 0.007755 and 0.118 respectively. The effect of variation of the second and third of these parameters, within their experimental uncertainies, on the CL for agreement of the data with

[10] This effect is particularly transparent in Fig. 1 of [18] or in Fig. 15.1 of [4]

[11] Indeed, the consistency of the three different 'hadronic' estimates of A_l is much better than expected. Because of the large statistical uncertainties of the c- and all-quark data this is most likely due to a chance co-incidence rather than any over-estimation of systematic errors.

[12] As mentioned in the Introduction, the measured value of m_t used in this section is the old pre-2004 one.

Table 7. Results of m_H fits to different sets of observables

Fitted quantities	m_H [GeV]	χ^2/d.o.f, CL
A_l(lept), $\overline{s}_l$,A_b, $\overline{s}_{\rm b}$, A_c, $\overline{s}_{\rm c}$, m_W	97^{+31}_{-24}	14.7/8, 0.065
A_l(all), m_W	97^{+32}_{-24}	1.99/1, 0.16
A_l(lept), $\overline{s}_l$,A_b, $\overline{s}_{\rm b}$, A_c, $\overline{s}_{\rm c}$, m_W m_W(NuTeV)	112^{+35}_{-27}	23.3/9, 0.0056
A_l(all), m_W, m_W(NuTeV)	113^{+36}_{-28}	10.6./2, 0.0050
A_l(lept), m_W	53^{+22}_{-18}	0.20/1, 0.66
A_l(lept), m_W, m_W(NuTeV)	66^{+28}_{-20}	10.9/2, 0.0043
A_l(had), m_W	154^{+65}_{-47}	8.0/1, 0.0047
A_l(had), m_W, m_W(NuTeV)	196^{+79}_{-58}	14.6/2, 0.00068

the SM, will be discussed in Sect. 5 below. The values of other fixed parameters are specified in [16–18].

In the fits, a numerical parameterisation, accurate at the per mil level, of the effective weak mixing angle given by the two-loop ZFITTER 5.10 program [33] was used[13]:

$$\sin^2\Theta^{\rm lept}_{\rm eff} = 0.233657 - 8.42\times10^{-8}m_t^2 - 3.86\times10^{-4}\ln m_t \\ +5.00\times10^{-4}\ln m_H \qquad (4.1)$$

where m_t and m_H are expressed in GeV units. The overall normalisation factors ρ_f $(f = l, \nu, u, d, b)$ for fermionic widths of the Z are given by a numerical parametrisation similar to (4.1) of the entries in Table 2 of [34]. For m_W, the parameterisation of [35] was used.

The fits for m_H are then repeated using only the 'm_H-sensitive' observables A_l(all) and m_W where A_l(all) is the weighted average of the leptonic and hadronic values of A_l given in the last column of Table 6. Similar fits are performed including also m_W(NuTeV). The results of this comparison are shown in the first four rows of Table 7. In can be seen that essentially the same range of Higgs masses is obtained whether fits are made to the complete set of electroweak observables or only to the m_H-sensitive ones A_l and m_W. The fit results presented in the fifth and sixth rows of Table 7 demonstrate that very low values of m_H, with best fit values incompatible with the 95% direct lower limit of 114.4 GeV, are found when fitting only the m_H-sensitive observables $A(\text{lept})_l$ and m_W. As shown in the last two rows of Table 7, much higher values of m_H are found when $A(\text{lept})_l$ is replaced by $A(\text{had})_l$ in the fits. This is a consequence of the deviation of the measured value of A_b from the SM expectation, and the strong correlation between A_l and A_b resulting from (3.1), when $A^{0,\rm b}_{\rm FB}$ is measured. In all cases inclusion of the NuTeV m_W measurement results in slightly higher fitted values for m_H and reduces all confidence levels, by about an order of magnitude, to values less than 0.01.

As another cross-check of both the fitting procedure and the m_H, m_t sensitivity of different observables, simultaneous fits to m_H and m_t were performed including also in the χ^2 estimator the directly measured value of m_t from FERMILAB: $m_t = 174.3(5.1)$ GeV. The results of these fits, both including and excluding the indirect NuTeV m_W measurement, are presented in Table 8, where they may be compared with the results of similar fits from the most recent EWWG report [4]. Slightly lower fitted values of m_H are found in the latter, probably due to the inclusion of other observables such as Γ_Z, Γ_W and $Q_W(Cs)$ from atomic parity-violating experiments, that have some sensitivity to m_H, in the EWWG fits. The uncertainties on both m_H and m_t found in the two sets of fits are very similar. In fact slightly more precise values of m_H are obtained in the fits of the present paper. This, in combination with the results shown in the first four rows of Table 7, shows that the restriction to A_l and m_W entails no significant loss of sensitivity in the indirect determination of m_H. The dilution effect discussed in Sect. 1 of the hypothesis-testing power of the χ^2 estimator, due to the inclusion of unaveraged equivalent obervables, or additional 'noise' observables, that are insensitive to m_H and m_t, is evident in the χ^2/d.o.fs and CLs of the fits that are also presented in Table 8. The EWWG fits have a CL that is a factor of 8(4) times larger than those of the present paper for the fits including(excluding) the NuTeV m_W measurement. A more detailed discussion of the global EWWG fits is found in Sect. 8 below.

Table 8. Global electroweak fits for m_H and m_t

		all data except NuTeV	all data
This	m_H	102^{+53}_{-35}	107^{+58}_{-37}
paper	m_t	$175.0^{+4.4}_{-4.2}$	$173.7^{+4.5}_{-4.3}$
	χ^2/d.o.f, CL	14.7/8, 0.065	23.3/9, 0.0056
EWWG	m_H	91^{+55}_{-36}	96^{+60}_{-38}
[4]	m_t	$175.3^{+4.4}_{-4.3}$	$174.3^{+4.5}_{-4.4}$
	χ^2/d.o.f, CL	16.7/14, 0.27	25.4/15, 0.045

5 Combining confidence levels of direct and indirect limits on m_H

The combined result of the direct searches for the Standard Model Higgs Boson by the LEP Collaborations

[13] The formula is valid at the quoted accuracy for $m_H \geq 40$ GeV. For lower values of m_H, small corrections are made to the constant term and the coefficient of $\ln m_H$.

Table 9. Confidence levels as a function of m_H for different sets of observables. The values of $\chi^2_{\rm WA}$/d.o.f. and CL refer to A_l

Observables	m_H [GeV]	120	160	200	240	280
A_l(all), m_W	CL($\chi^2_{\rm SM}$)	0.51	0.35	0.13	0.047	0.0074
$\chi^2_{\rm WA}$/d.o.f. = 9.0/3	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.065	0.049	0.022	0.0073	0.0021
CL = 0.029	CL(*Comb*)	0.076	0.057	0.024	0.0104	0.0020
A_l(lept), m_W	CL($\chi^2_{\rm SM}$)	0.30	0.054	0.008	0.0011	0.00017
$\chi^2_{\rm WA}$/d.o.f. = 1.6/2	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.41 0	0.11	0.024	0.0044	0.00079
CL = 0.45	CL(*Comb*)	0.41 0	0.11	0.024	0.0044	0.00079
A_l(had), m_W	CL($\chi^2_{\rm SM}$)	0.11	0.024	0.029	0.026	0.019
$\chi^2_{\rm WA}$/d.o.f. = 0.047/2	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.058	0.111	0.130	0.118	0.091
CL = 0.98	CL(*Comb*)	0.058	0.111	0.130	0.118	0.091

Table 10. Confidence levels as a function of m_H for different sets of observables

Observables	m_H [GeV]	120	160	200	240	280
A_l(all), m_W,	CL($\chi^2_{\rm SM}$)	0.012	0.015	0.0089	0.0036	0.0012
m_W(NuTeV)	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.0027	0.0033	0.0021	0.00096	0.00036
	CL(*Comb*)	0.0030	0.0037	0.0024	0.00107	0.00039
A_l(lept), m_W,	CL($\chi^2_{\rm SM}$)	0.0071	0.0025	0.00066	0.00013	3.3×10^{-5}
m_W(NuTeV)	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.018	0.0072	0.0022	0.00056	0.00014
	CL(*Comb*)	0.022	0.0089	0.0027	0.00065	0.00018
A_l(had), m_W,	CL($\chi^2_{\rm SM}$)	0.00031	0.0012	0.0022	0.0029	0.0029
m_W(NuTeV)	CL($\chi^2_{\rm SM} + \chi^2_{\rm WA}$)	0.0021	0.007	0.012	0.015	0.015
	CL(*Comb*)	0.0028	0.0091	0.016	0.020	0.019

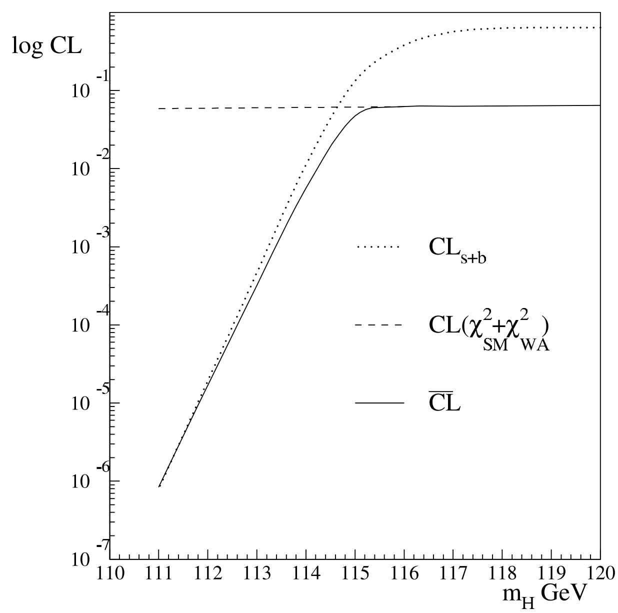

Fig. 1. Illustration of the combination of direct (CL_{s+b}) and indirect $\mathrm{CL}(\chi^2_{\rm SM} + \chi^2_{\rm WA})$ m_H confidence levels using (2.8). The observables used to calculate $\mathrm{CL}(\chi^2_{\rm SM} + \chi^2_{\rm WA})$ are A_l(all) and m_W

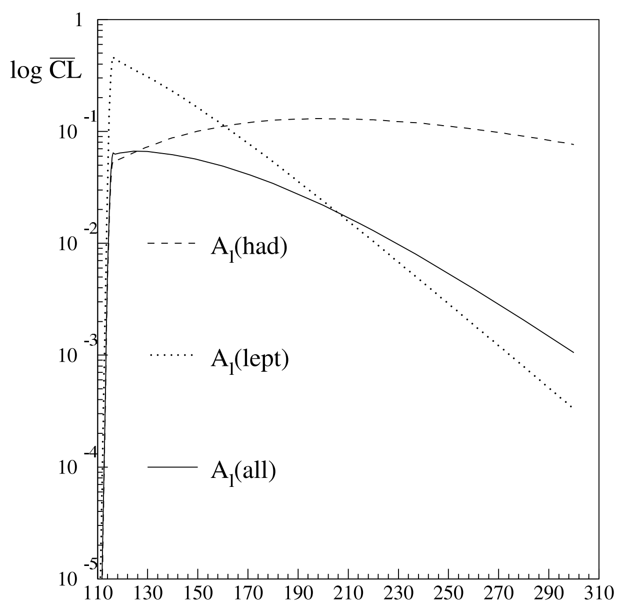

Fig. 2. Combined m_H confidence levels. The observables used to calculate $\mathrm{CL}(\chi^2_{\rm SM} + \chi^2_{\rm WA})$ are A_l and m_W

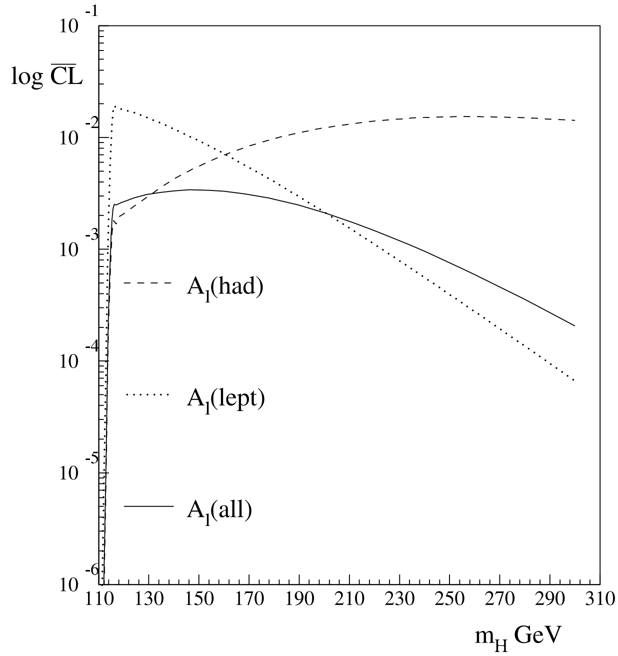

Fig. 3. Combined m_H confidence levels. The observables used to calculate $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ are A_l, m_W and m_W(NuTeV)

Table 11. Combined confidence levels $\overline{CL}$ for consistency with the SM as a function of m_H. Observables used in the χ^2 estimator: A_l and m_W

	A_l(All)	A_l(lept)	A_l(had)
m_H (GeV)			
111	8.4×10^{-7}	6.3×10^{-6}	6.6×10^{-7}
113	3.2×10^{-4}	2.1×10^{-3}	2.6×10^{-4}
115	0.047	0.23	0.041
140	0.062	0.23	0.088
180	0.034	0.054	0.13
220	0.013	0.010	0.13
260	3.9×10^{-3}	1.9×10^{-3}	0.11
300	1.1×10^{-3}	3.3×10^{-4}	0.077

Table 12. Combined confidence levels $\overline{CL}$ for consistency with the SM as a function of m_H. Observables used in the χ^2 estimator: A_l, m_W and m_W(NuTeV)

	A_l(All)	A_l(lept)	A_l(had)
m_H (GeV)			
111	3.6×10^{-8}	3.0×10^{-7}	2.6×10^{-8}
113	1.6×10^{-5}	1.1×10^{-4}	1.1×10^{-5}
115	2.8×10^{-3}	0.017	2.2×10^{-3}
140	3.3×10^{-3}	0.012	4.2×10^{-3}
180	2.8×10^{-3}	4.0×10^{-3}	9.8×10^{-3}
220	1.5×10^{-3}	1.1×10^{-3}	0.014
260	6.0×10^{-4}	2.8×10^{-4}	0.015
300	2.1×10^{-4}	6.6×10^{-5}	0.014

ALEPH, DELPHI, L3 and OPAL is given in Fig. 9 of [10]. This shows the confidence level ratio: $\mathrm{CL}_s \equiv \mathrm{CL}_{s+b}/\mathrm{CL}_b$ as a function of m_H. CL_{s+b} is the confidence level of the signal-plus-background hypothesis and CL_b that of the background-only hypothesis. Inspection of the figure shows that, at percent level accuracy: $\mathrm{CL}_s = 10^{-6}, 0.05, 0.08$ for $m_H = 111, 114.4, 120$ GeV, respectively. For the present study it is preferred to work directly with CL_{s+b}, which is similar to the χ^2 confidence level given by comparing the SM to Z-decay data, to obtain indirect m_H limits. As shown in Fig. 7 of [10], the value of CL_b is about 0.8 in the region 110 GeV $< m_H <$ 120 GeV, of interest for the present study. This gives the estimates: $\mathrm{CL}_{s+b} = 10^{-7}, 0.04, 0.64$ for $m_H = 111, 114.4, 120$GeV, respectively.

The following numerical parameterisation of CL_{s+b} is used:

111 GeV $< m_H <$ 114.4 GeV

$$\log \mathrm{CL}_{s+b} = 1.382 m_H(\mathrm{GeV}) - 159.51 \quad (5.1)$$

114.4 GeV $\le m_H <$ 120 GeV

$$\log \mathrm{CL}_{s+b} = -0.1938 - \left(\frac{120 - m_H(\mathrm{GeV})}{5.3945}\right)^{4.968} \quad (5.2)$$

As shown in Fig. 1, the function of (5.2) has the same value and first derivative as that of (5.1), at the matching point $m_H = 114.4$ GeV, and vanishing first derivative at $m_H = 120$ GeV, where $\mathrm{CL}_{s+b} = 0.64$. Allowing for the overall scale factor of 0.8, the parameterisation of (5.1) and (5.2) describes well the experimentally determined curve of $\mathrm{CL}_{s+b}/\mathrm{CL}_b$ in Fig. 9 of [10]. It should be noted, however, that the precise shape of CL_{s+b} has only a small effect on the final confidence level curves to be presented below. The direct search excludes, with a CL of $\le 10^{-3}$, the possibility that the SM Higgs boson exists with mass of less than 113 GeV, and gives essentially no information for $m_H > 115$ GeV. Thus the region where it is of interest to combine CL_{s+b} with indirect confidence levels covers only a narrow range of m_H.

In order to define the confidence level for agreement of Z-decay data with the SM, the χ^2 of the data/SM comparision is simply calculated as a function of m_H, setting m_t and $\alpha(m_Z)$ to the measured values given above. Therefore no fit to the data is necessary. The sensitivity of the CL curves to the assumed values of m_t and $\alpha(m_Z)$ is discussed below. In this section only the 'm_H-sensitive' observables A_l and m_W are included in the χ^2 estimator, where the W mass is either the directly measured value from LEP and FERMILAB or the indirectly determined NuTeV value. A_l is determined either by using all asymmety data (A_l(all)), lepton data only (A_l(lept)) or only hadronic data (A_l(had)). The corresponding values are presented in Table 6 above. To take into account the internal consistency of the different data sets the values of $\chi^2_{\mathrm{all,WA}}$, $\chi^2_{\mathrm{lept,WA}}$, or $\chi^2_{\mathrm{had,WA}}$ are added to the χ^2 of the SM comparison: $\chi^2_{\mathrm{WA,SM}}$ in each case. As shown below, almost identical CLs are found using either $\chi^2_{X,WA} + \chi^2_{\mathrm{WA,SM}}$ ($X = $ all, lept, had) or by combining the CLs of $\chi^2_{X,WA}$ and $\chi^2_{\mathrm{WA,SM}}$ using (2.8). The former CL is then combined with CL_{s+b} using (2.8) to yield the direct plus indirect confidence level curves shown below. The values of $\chi^2/\mathrm{d.o.f.}$ for $\chi^2_{\mathrm{had,WA}}$ and $\chi^2_{\mathrm{all,WA}}$ are given above; that for the leptonic data: $\chi^2_{\mathrm{lept,WA}}/\mathrm{d.o.f.} = 1.6/2$, CL $= 0.45$, given by combining the A_l values obtained from lepton forward/backward asymmetries and tau polarisation measurements from LEP and the A_{LR} measurement from SLD, is taken from [4].

Some typical CLs for the indirect m_H analysis, obtained as described above, are presented in Table 9 (observables considered: A_l, m_W) and Table 10 (observables considered: A_l, m_W, m_W(NuTeV)). In all cases good agreement is found between $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ and $\mathrm{CL}(Comb)$ calculated using (2.8), where the abbreviations $\chi^2_{\mathrm{SM}} \equiv \chi^2_{\mathrm{WA,SM}}$, $\chi^2_{\mathrm{WA}} \equiv \chi^2_{X,WA}$ have been introduced.

The combination of $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ (indirect measurements) and CL_{s+b} (direct measurements) for different values of m_H is illustrated in Fig. 1. Since CL_{s+b} provides little information on m_H for $m_H > 114.4$ GeV (the 95% CL lower limit of the direct search), $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ is combined with CL_{s+b} provided that $\mathrm{CL}(Comb)$ is less than $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$. In the contrary case $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ alone is used. Thus the algorithm used to obtain the combined confidence level, $\overline{\mathrm{CL}}$, is:

- Calculate α_3 from $\alpha_1 = \mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ and $\alpha_2 = \mathrm{CL}_{s+b}$ according to (2.8).
- If $\alpha_3 < \mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$, set $\overline{\mathrm{CL}} = \mathrm{CL}(\alpha_1\ \alpha_2)$.
- If $\alpha_3 \geq \mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$, set $\overline{\mathrm{CL}} = \mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$.

In Fig. 1 the dotted curve shows CL_{s+b}, the dashed curve $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ and the solid curve $\overline{\mathrm{CL}}$.

Curves of $\overline{\mathrm{CL}}$ calculated in this manner are shown in Figs. 2 and 3. When A_l(lept) and A_l(all) are used, the general shape, with a sharp peak above, but close to, the direct lower limit and a rapid fall-off for higher values of m_H is similar to that of the PDFs presented in [22,23]. However $\overline{\mathrm{CL}}$, unlike the PDFs, gives an estimate of the absolute probability that the data is consistent with the SM for a given value of m_H. An exception to this behaviour is provided by the data sets A_l(had), m_W and A_l(had), m_W, m_W(NuTeV) where the maximum of $\overline{\mathrm{CL}}$ occurs at much higher values of m_H and on average much larger values of $\overline{\mathrm{CL}}$ are obtained. The confidence level curves in Figs. 2 and 3 are presented in numerical form in Tables 11 and 12, in the same format as the similar curves considered in Sect. 7 below. The latter use all precision observables rather than only the m_H-sensitive ones as in Figs. 2 and 3. Comparison of the two sets of curves then shows the effect of 'non m_H-sensitive' observables on the level of agreement with the SM prediction.

The effect on the $\overline{\mathrm{CL}}$ curves of variation of the value of m_t by plus or minus the experimental error around the measured value is shown in Fig. 4 and Table 13. The effect of a similar variation of $\alpha(m_Z)$ is shown in Fig. 5 and Table 13. For large values of m_H this variation of m_t changes the values of $\overline{\mathrm{CL}}$ by many orders of magnitude. Even so, in the case of the data set A_l(lept), m_W, shown in Figs. 4 and 5, which can be argued (see below) to be likely to give the most reliable estimate of m_H, $\overline{\mathrm{CL}}$ is still only, at best, 0.01, at $m_H = 300$ GeV. Change of $\alpha(m_Z)$ within the current experimental errors can change $\overline{\mathrm{CL}}$ by up to an order of magnitude, but the effect is much less dramatic than for m_t. Clearly a much improved measurement of m_t is needed to significantly improve the indirect limits on m_H.

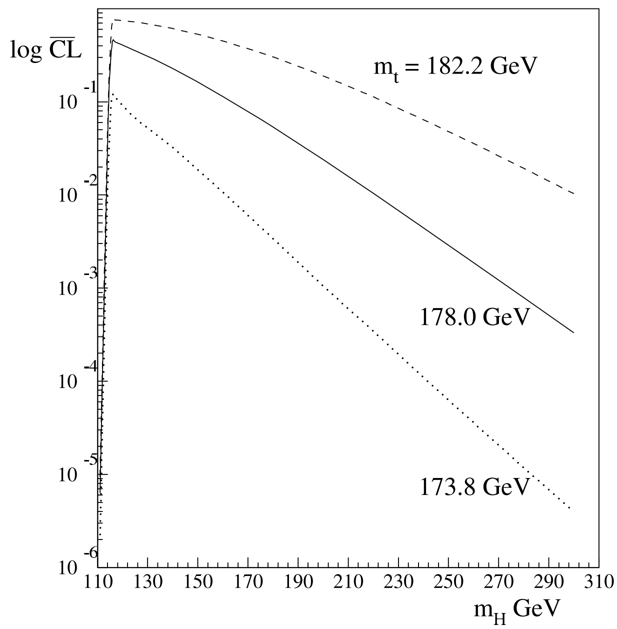

Fig. 4. Dependence of combined m_H confidence levels on the value of m_t. The observables used to calculate $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ are A_l(lept) and m_W. $\alpha(m_Z) = 0.007755$ is assumed

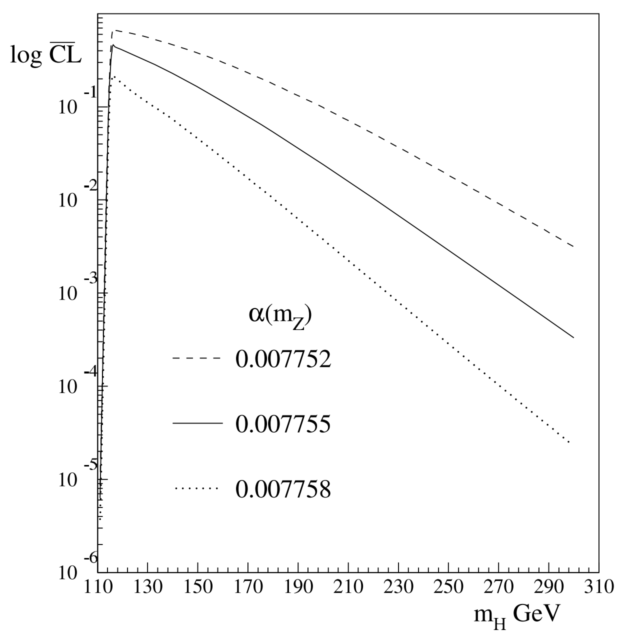

Fig. 5. Dependence of combined m_H confidence levels on the value of $\alpha(m_Z)$. The observables used to calculate $\mathrm{CL}(\chi^2_{\mathrm{SM}} + \chi^2_{\mathrm{WA}})$ are A_l(lept) and m_W. $m_t = 178$ GeV is assumed

Table 13. Combined confidence level curves $\overline{\mathrm{CL}}$ for variation of m_t and $\alpha(m_Z)$ by plus or minus one standard deviation around their measured values. Observables used in the χ^2 estimator: A_l(lept) and m_W

	$\alpha(m_Z) = 0.007755$		$m_t = 178$ GeV	
m_H GeV	$m_t = 173.8$ GeV	182.2 GeV	$\alpha(m_Z) = 0.007752$	0.007758
111	2.1×10^{-6}	9.4×10^{-6}	8.6×10^{-6}	3.5×10^{-6}
113	6.9×10^{-4}	3.2×10^{-3}	2.9×10^{-3}	1.1×10^{-3}
115	0.085	0.33	0.30	0.13
140	0.032	0.61	0.46	0.073
180	3.4×10^{-3}	0.30	0.18	0.010
220	3.5×10^{-4}	0.11	0.051	1.3×10^{-3}
260	3.6×10^{-5}	0.036	0.013	1.7×10^{-4}
300	4.0×10^{-6}	0.010	3.1×10^{-3}	2.3×10^{-5}

The analysis presented in this section has many similarities with that of [19] where confidence levels taking into account both direct and indirect information on m_H were derived. However, the present writer has doubts about the mathematical correctness of the method used in [19]. It consists of combining the confidence level given by the χ^2_{min} and *d.o.f.* of a fit with that derived from $\Delta\chi^2 = \chi^2(m_H = 114 \text{ GeV}) - \chi^2_{\mathrm{min}}$ of the *same* fit, assuming them to be independent. Certainly, the formula used to combine the confidence levels, simple multiplication instead of (2.8) above, is incorrect[14]. The approach used is essentially to replace the measured CL_{s+b} curve by a θ-function at $m_H = 114$ GeV. As can be seen in Fig. 1, this is quite a good approximation. The global fits used in [19] took no account of the dilution of the hypothesis testing power of the χ^2 estimator resulting from the use of unaveraged and insensitive observables, as discussed in Sect. 2 above and, in more detail, in Sect. 8 below. For instance there is the statement: 'The global SM fit was excellent in 1998 and has now (2002) become poor'. This is not at all true of the contribution to the χ^2 of the m_H-sensitive observables, which is similarly high for both data sets. In fact, the high confidence level for the 1998 global fit is a consequence of an anomalously *low* contribution from the non m_H-sensitive observables [18]. Also, although the NuTeV measurement was discussed, together with A_b, as a possible source of anomaly relative to the SM prediction, only the published interpretation as a measure of $\sin^2\theta_W^{\mathrm{on-shell}}$ or m_W was considered. Also the correctness of the SM prediction for the quark couplings to the Z was assumed to define different values of $\sin^2\Theta_{\mathrm{eff}}^{\mathrm{lept}}$ (equivalent to A_l in the present paper). The same two assumptions have been made in the analysis presented in this section. In the analysis of model-independent observables presented in Sect. 7 below an alternative interpretation of the NuTeV result, discussed in the following section, as a measurement of the $\mathrm{Z}\nu\overline{\nu}$ coupling is also used.

An important point stressed in [19], first pointed out in [18], is that, regardless of how the A_b anomaly is interpreted (statistical fluctuation, unknown systematic effect or new physics) the most reliable estimate of m_H must be that derived from the charged lepton asymmetries and m_W that give consistent predictions for this quantity. Inclusion of the $A_{\mathrm{FB}}^{0,\mathrm{b}}$ measurements in the fit results in a positive $\simeq 50$ GeV bias on the 95% CL upper limit on m_H, due to the A_l-A_b correlation resulting from (3.1). Although fits with and without the 'anomalous' NuTeV measurement are routinely presented by the EWWG, fits excluding the (equally 'anomalous') hadron asymmetry data that, would provide the most reliable estimate of m_H, have (to my best knowledge) never been shown in the periodical updates of the status of electroweak measurements, such as [4], produced by this working group. I agree with almost all of the general conclusions of [19], in particular, that if the lepton asymmetry data is correct then, regardless of the status of the hadronic asymmetry data, the SM provides only a poor global description of the data. I would also remark that the confidence levels of the global fits quoted, although small, will become even smaller when corrected for the dilution effects discussed in Sect. 2 above. Use of the correct formula for combining confidence levels will, on the other hand, give higher combined confidence levels. The analysis of [19] gives, however, no hint of the very low values of $\overline{\mathrm{CL}}$ for large values of m_H apparent in Figs. 2 and 3 and Tables 11 and 12. Finally, in connection with [19], as discussed in Sect. 8 below, the use of $\Delta\chi^2$ to provide confidence levels for parameter estimation is of doubtful validity when, as is the case for the current electroweak data, the absolute confidence level derived from χ^2_{min} shows that the model containing the parameter of interest does not adequately describe the data.

6 Alternative interpretations of the NuTeV experiment

The publication of the results of the NuTeV experiment [32] gives an estimation of the value of the on-shell weak mixing angle:

$$\sin^2\theta_W^{\mathrm{on-shell}} = 0.2277(13)(9) \tag{6.1}$$

[14] The present author made the same mistake in [16]

that may be translated directly into a W-mass measurement via the defining relation of the on-shell renormalisation scheme:

$$\sin^2\theta_W^{\rm on-shell} \equiv 1 - \frac{m_W^2}{m_Z^2} \qquad (6.2)$$

This is done by use of the Paschos-Wolfenstein relation [36]:

$$R_- = \frac{\sigma_{NC}(\nu)-\sigma_{NC}(\overline{\nu})}{\sigma_{NC}(\nu)+\sigma_{NC}(\overline{\nu})} = 4(\overline{g}_\nu^L)^2 \sum_{q=u,d}[(\overline{g}_{\rm q}^L)^2-(\overline{g}_{\rm q}^R)^2]$$
$$= \rho_\nu\rho_{\rm ud}[\frac{1}{2}-\sin^2\theta_W^{\rm on-shell}] \qquad (6.3)$$

The above interpretation however requires that the parameters ρ_ν and $\rho_{\rm ud}$ are assigned their standard model values. As discussed in [37] and shown, for example, in Fig. 1 of [38], the experiment actually measures the quantity on the right side of (6.3) that is sensitive both to $\sin^2\theta_W^{\rm on-shell}$ and to $\rho_\nu\rho_{\rm ud}$. The latter quantity may be specified by a parameter ρ_0 such that:

$$\rho_\nu\rho_{\rm ud} = \rho_0^2\rho_\nu(SM)\rho_{\rm ud}(SM) \qquad (6.4)$$

The good agreement with the SM prediction for the quantities $\Gamma_{\rm had}$ (0.03%, 0.25σ deviation). $\overline{s}_{\rm c}$ (0.5 %, 0.3σ deviation) and $\overline{s}'_{\rm nb}$ (0.05%, 0.16σ deviation), to be discussed in the following section, gives a strong indication that, at the per mille level, $\rho_{\rm ud} \simeq \rho_{\rm ud}(SM)$. So that, to this accuracy (6.4) simplifies to:

$$\rho_\nu = \rho_0^2\rho_\nu(SM) \qquad (6.5)$$

or, equivalently,

$$\overline{s}_\nu = \rho_0^2\overline{s}_\nu(SM) \qquad (6.6)$$

Assuming the SM value ($\rho_0 = 1$) gives a prediction for $\sin^2\theta_W^{\rm on-shell}$:

$$\sin^2\theta_W^{\rm on-shell} = 0.22733(135)(93)$$
$$-8.8\times10^{-8}(m_t[{\rm GeV}]^2-175^2)$$
$$+3.2\times10^{-4}\ln(m_H[{\rm GeV}]/150) \qquad (6.7)$$

Assuming instead the SM value of $\sin^2\theta_W^{\rm on-shell}$ the experiment provides a measurement of ρ_0 [37]:

$$\rho_0 = 0.9942(13)(16) + 2.4\times10^{-8}(m_t[{\rm GeV}]^2-175^2)$$
$$-1.6\times10^{-4}\ln(m_H[{\rm GeV}]/150) \qquad (6.8)$$

Choosing the values $m_t = 175$ GeV and $m_H = 150$ GeV consistent with the measured value of m_W, (6.6) and (6.8) give a measurement of the model-independent parameter $\overline{s}_\nu$:

$$\overline{s}_\nu({\rm NuTeV}) = 0.4992(21) \qquad (6.9)$$

which may be compared with the LEP measurement quoted in Table 5:

$$\overline{s}_\nu({\rm LEP}) = 0.5014(15) \qquad (6.10)$$

Since:

$$\overline{s}_\nu({\rm LEP}) - \overline{s}_\nu({\rm NuTeV}) = 0.0022(26) \qquad (6.11)$$

the two measurements are quite consistent ($\chi^2_{{\rm WA},\overline{s}_\nu}/d.o.f. = 0.78/1$, CL = 0.38) and yield the weighted average value:

$$\overline{s}_\nu({\rm LEP+NuTeV}) = 0.5006(12) \qquad (6.12)$$

which differs from the SM prediction (see Table 14) by 0.9% and 3.7σ. Note that this deviation is much larger than that of the quantities $\Gamma_{\rm had}$, $\overline{s}'_{\rm nb}$ and $\overline{s}_{\rm c}$ which suggests that, as assumed here, ρ_ν and not $\rho_{\rm ud}$ is most likely the source of the anomalous behaviour of (6.3). Alternatively assuming that $\rho_0 = 1$ and using (6.2) and (6.7) to obtain m_W gives:

$$\overline{s}_\nu({\rm NuTeV}) = \overline{s}_\nu({\rm SM}) = 0.5050 \qquad (6.13)$$
$$m_W({\rm NuTeV}) = 80.136(83)\ {\rm GeV} \qquad (6.14)$$

In this case the assumed, SM, value of $\overline{s}_\nu$ differs from the LEP measurement by 2.4σ ($\chi^2/d.o.f. = 5.76/1$, CL = 0.016) and also

$$m_W({\rm LEP+FERMILAB}) - m_W({\rm NuTeV})$$
$$= 0.290(90) \qquad (6.15)$$

the 3.2σ discrepancy ($\chi^2/d.o.f. = 10.4/1$, CL = 0.0013) mentioned in [32]. Using (2.8) to combine the data consistency CLs for $\overline{s}_\nu$ and m_W yields an overall CL of 2.5×10^{-4}. Thus on the assumption that the NuTeV measurement is correct, the alternative interpretation of the experiment is strongly favoured statistically as the ratio of data consistency CLs of the two interpretations is $\simeq 1.5\times10^3$.

For both interpretations the SM prediction is unfavoured. For the standard one (m_W measurement and $\rho_0 = 1$) the CL of the SM comparison is that just quoted: 2.5×10^{-4}. For the alternative interpretation ($\rho_0 \neq 1$) it is found that:

$$\overline{s}_\nu({\rm LEP+NuTeV}) - \overline{s}_\nu({\rm SM}) = -0.0044(12) \qquad (6.16)$$

also a 3.7σ ($\chi^2/d.o.f. = 13.4/1$, CL = 2.5×10^{-4}) deviation from the SM prediction. The alternative interpretation thus shows exactly the same deviation from the SM as the one proposed in [32].

A number of theoretical assumptions must be made in order to derive (6.7) and (6.8) from the experimental quantities:

$$R_\nu^{\rm exp} = \frac{\sigma(\nu Fe\to\nu X)}{\sigma(\nu Fe\to\mu^- X)},\quad R_{\overline{\nu}}^{\rm exp} = \frac{\sigma(\overline{\nu}Fe\to\overline{\nu}X)}{\sigma(\overline{\nu}Fe\to\mu^+ X)}$$

actually measured by the NuTeV experiment. A recent concise review of the situation may be found in [20] in which citations of related work can be found. The most important and extensively discussed assumption concerns the supposed symmetry of the strange sea momentum distribution in a nucleon. A recent analysis by the CTEQ

Collaboration [39] presented in [20] finds some evidence for a positive asymmetry of the strange quark sea:

$$s^- = \int_0^1 x(s(x) - \overline{s}(x))dx = 0.002(1) \tag{6.17}$$

It is pointed out in [20] that an asymmetry of 0.002 has the effect of reducing by 42% the discrepancy between the measured value of $\sin^2\theta_W^{\mathrm{on-shell}}$ derived from the direct m_W measurements, and the value of the same quantity found from (6.7). The alternative, and statistically favoured, interpretation of the NuTeV experiment as a measurement of ρ_0, was not considered in [20], but in view of the linear correlation between $\sin^2\theta_W^{\mathrm{on-shell}}$ and ρ_0 provided by the measurement[15], it is reasonable to suppose that, in the alternative interpretation, the strange quark sea asymmetry will reduce the deviation of ρ_0 (or, equivalently $\overline{s}_\nu$) from the SM expectation by the same fraction. Thus the estimated value of $\overline{s}_\nu$, correcting for the effect of the asymmetry of (6.17) is 0.5015(21), which agrees prefectly with LEP measurement in (6.10). The LEP+NuTeV weighted average becomes 0.5014(12), which still lies 3.0σ below the SM expectation. The apparent anomaly in the $\mathrm{Z}\nu\overline{\nu}$ coupling is therefore reduced, but not removed, by the estimated effect of a strange quark sea asymmetry on the NuTeV results.

It remains true however that because of the many systematic effects, detailed in [20], the results of the NuTeV experiments are less 'sure' than the measurement of the related quantity Γ_{inv} at LEP. Because of this some authors [40] prefer to adopt a conservative position and exclude the NuTeV results completely from global electroweak analyses. In contrast, the present paper takes a strictly neutral position on the question of the reliability, or otherwise, of the NuTeV results. In the following section then, CLs as a function of m_H will be calculated using all available LEP and SLD data on the assumption of either of the two possible interpretations of the NuTeV experiment, or by excluding the experiment. The CLs obtained are later compared in Sect. 8 below with those obtained from the global EWWG and EWPDG fits.

7 Model-independent observables compared to SM predictions

Following the model-independent approach of [16–18] essentially all precision information on the Higgs sector of the SM provided, to date, by the LEP, SLC and FERMILAB experimental programs, as well as by the NuTeV experiment, is contained in the values of the nine observables listed in Table 14. The other important quantities m_t and $\alpha(m_Z)$ are not included as they are here considered as input parameters for the SM prediction rather than measurements which provide a test of the SM. All information from leptonic forward/backward charge asymmetry and τ-polarisation measurements as well as the SLC ALR measurement is condensed into the single parameter A_l(lept), equivalent to $\sin^2\Theta_{\mathrm{eff}}^{\mathrm{lept}}$. The quantity $\overline{s}_l$ is derived from the width of the Z for decay into charged leptons on the assumption of charged-lepton universality. The quantities A_c and A_b are derived using (3.1) from the c- and b-quark forward/backward charge asymmetries measured at LEP as well as from the SLC measurement of forward/backward-left/right asymmetries of c and b quarks. $\overline{s}_{\mathrm{c}}$ and $\overline{s}_{\mathrm{b}}$ are obtained from the Z decay widths into c and b quarks, more conventionally expressed in terms of the ratios: $R_Q = \Gamma_Q/\Gamma_{\mathrm{had}}$, $(Q = c, b)$, using the relation:

$$\overline{s}_Q = \sqrt{\frac{2\pi}{3}\frac{R_Q\Gamma_Z}{G_\mu M_Z^2}\frac{\sqrt{R_l\sigma_h^0}}{C_Q^{\mathrm{QED}}C_Q^{\mathrm{QCD}}}} \quad (Q = c, b), \tag{7.1}$$

The QED and QCD correction factors are given in [16]. The observable $\overline{s}_\nu$ is given by the invisible width, Γ_{inv}, of the Z boson, determined from the Z-boson total width, Γ_Z, the hadronic width, Γ_{had}, and the leptonic width, Γ_l via the relation:

$$\Gamma_{\mathrm{inv}} = \Gamma_Z - \Gamma_{\mathrm{had}} - 3\Gamma_l \tag{7.2}$$

Since the hadronic width of the Z is quite precisely measured: $\Gamma_{\mathrm{had}} = 1.7444(20)$ GeV, the measurements of R_b or $\overline{s}_{\mathrm{b}}$ can be used, in combination with the former, to extract the quantity:

$$\overline{s}'_{\mathrm{nb}} = \sum_{q=u,c}[(\overline{v}_q)^2 + (\overline{a}_q)^2]C_u^{\mathrm{QED}} + \sum_{q=d,s}[(\overline{v}_q)^2 + (\overline{a}_q)^2]C_d^{\mathrm{QED}} \tag{7.3}$$

The subscript 'nb' here stands for 'non-b' quarks. As will be discussed below, the measurements of Γ_{had} and $\overline{s}'_{\mathrm{nb}}$ provide much more stringent constraints on the possible values of the couplings of non-b down-type quarks to the Z than the existing direct measurements of these couplings, which have large experimental errors.

The experimental errors on the observables listed in Table 14 are largely uncorrelated between the observables, which facilitates calculation of a χ^2 estimator for global SM comparisons. Correlations exist between: A_l, A_c(LEP) and A_b(LEP) due to the use of (3.1) to obtain A_c(LEP) and A_b(LEP). Because of the small uncertainty on Γ_{had} the errors on $\overline{s}_{\mathrm{b}}$ and $\overline{s}'_{\mathrm{nb}}$ are strongly anticorrelated. Weaker correlations exist between $\overline{s}_{\mathrm{c}}$ and $\overline{s}_{\mathrm{b}}$. In view of the relatively poor precision of the $\overline{s}_{\mathrm{c}}$ measurement in comparison with those of $\overline{s}_{\mathrm{b}}$ and $\overline{s}'_{\mathrm{nb}}$, and the correlations between these three observables, the contribution of the former is omitted from the χ^2 estimator used in the global comparisons with the SM shown below. As previously mentioned, to take properly into account correlations, the direct SLC measurements of A_c and A_b are assigned separate terms from the LEP measurements in the χ^2 estimator. In Table 14, however, the weighted average LEP+SLC values of A_c and A_b are quoted.

The value of $\overline{s}_\nu$ given in Table 14 is the LEP+NuTeV weighted average, i.e. the statistically preferred 'alternative' interpretation of the NuTeV experiment is taken.

[15] See, for example, Fig. 1 of [38].

Table 14. Model-independent electroweak observables. The SM predictions correspond to $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_Z) = 0.007755$

X	$X_{\rm expt}$	$\sigma_X/X_{\rm expt}$ (%)	$X_{\rm SM}$	$(X_{\rm expt} - X_{\rm SM})/\sigma_X$
A_l(lept)	0.1501(16)	1.07	0.1481	1.25
$\overline{s}_l$	0.25268(26)	0.103	0.25277	−0.35
A_c	0.653(20)	3.06	0.668	−0.75
$\overline{s}_{\rm c}$	0.2897(50)	1.73	0.2884	0.26
A_b	0.902(13)	1.44	0.9347	−2.52
$\overline{s}_{\rm b}$	0.3663(13)	0.35	0.3648	1.15
$\overline{s}_\nu$	0.5006(12)	0.24	0.5050	−3.67
$\overline{s}'_{\rm nb}$	1.3211(43)	0.33	1.3218	−0.16
m_W	80.426(34)	0.042	80.394	0.94

Table 15. Effective vector and axial-vector coupling constants of the Z boson to lepton, neutrino, and heavy quark pairs. The SM predictions correspond to $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_Z) = 0.007755$

Coupling	Expt value	SM	(Exp-SM)/σ
$\overline{v}_l$	−0.03783(41)	−0.03734	−1.2
$\overline{a}_l$	−0.50125(26)	−0.50137	0.46
$\lvert\overline{v}_\nu\rvert = \lvert\overline{a}_\nu\rvert$	0.5003(6)	0.50251	−3.7
$\overline{v}_{\rm c}$	0.1875(69)	0.1921	−0.67
$\overline{a}_{\rm c}$	0.5045(50)	0.5015	0.60
$\overline{v}_{\rm b}$	−0.3232(78)	−0.3435	2.6
$\overline{a}_{\rm b}$	−0.5133(50)	−0.4983	−3.0

Table 16. History of measurements of $\overline{g}_{\rm b}^R$ and $\overline{g}_{\rm b}^L$. The SM predictions correspond to $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_Z) = 0.007755$

	Coupling	Value	(Exp-SM)/σ
SM	$\overline{g}_{\rm b}^R$	0.0774	–
	$\overline{g}_{\rm b}^L$	−0.4209	–
1996 data [30]	$\overline{g}_{\rm b}^R$	0.1098(101)	3.2
	$\overline{g}_{\rm b}^L$	−0.4155(30)	1.8
1998 data [41]	$\overline{g}_{\rm b}^R$	0.1050(90)	3.1
	$\overline{g}_{\rm b}^L$	−0.4159(24)	2.1
2003 data [4]	$\overline{g}_{\rm b}^R$	0.0951(63)	2.8
	$\overline{g}_{\rm b}^L$	−0.4182(16)	1.7

Table 17. Observables in the 1996 and 2003 data sets contributing to measurements of the b-quark effective coupling constants

X	1996	2003	$[X(2003)-X(1996)]/\sigma_X(2003)$
$A_{\rm FB}^{0,{\rm b}}$(LEP)	0.0979(23)	0.0997(16)	1.1
$\overline{s}_{\rm b}$(LEP)	0.3676(24)	0.3663(13)	−1.0
A_b(SLC)	0.863(49)	0.925(20)	3.1
A_l(LEP)	0.1466(33)	0.1482(26)	0.62
A_l(SLC)	0.1543(37)	0.1513(21)	−1.4
A_l(LEP+SLC)	0.1501(24)	0.1501(16)	0.0

The NuTeV measurement may be compared, in this sense, with the LEP measurements of $A_{\rm FB}^{0,{\rm Q}}$ (q=b,c). The latter depend, via (3.1), on A_l (defined by the values of the Z charged-lepton couplings) and A_Q (defined by the values of the Z heavy-quark couplings), Since A_l is independently measured, A_Q can then be extracted from the measured value of $A_{\rm FB}^{0,{\rm Q}}$. Similarly the NuTeV result depends, in a correlated way, on the values of $\sin^2\theta_W^{\rm on-shell}$ (equivalent to m_W) and ρ_0 (equivalent, on the assumption, consistent with the data, that ρ_{ud} is in agreement with the SM, to the Z$\nu\overline{\nu}$ coupling). Since m_W (and hence $\sin^2\theta_W^{\rm on-shell}$) is precisely determined at LEP and FERMILAB the correlated value of ρ_0, and so also $\overline{s}_\nu$, can be extracted in a similar fashion to A_Q from $A_{\rm FB}^{0,{\rm Q}}$.

The experimental values of the observables in Table 14 are compared with the SM prediction for $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_Z) = 0.007755$. This choice of m_H (just above the experimental lower limit) is near to the maxima of the A_l(lept) and A_l(all) curves of $\log\overline{\rm CL}$ plotted in Figs. 2 and 3. Note that the model-independent analysis corresponds to only the A_l(lept) curves in Figs. 2–6. Only by making the stronger assumption of the SM values of the Z couplings to quarks, is it possible to derive A_l(had) and A_l(all).

The agreement with the SM predictions shown in Table 14 is not completely satisfactory. The largest deviations are for $\overline{s}_\nu$ (−0.84% and 3.7σ) and A_b (−3.5% and 2.5σ). The positive 1.05σ and 0.94σ deviations of both A_l and m_W respectively reflect the fact that the central values of m_H preferred by these observables [16]:

$$m_H = 72.5^{+36.4}_{-24.2} \quad \text{fit of } A_l(\text{lept}) \text{ only}$$

$$m_H = 65.3^{+59.9}_{-37.8} \quad \text{fit of } m_W \text{ only}$$

are incompatible with the direct lower limit of $m_H = 114.4$ GeV.

The effective vector and axial vector couplings of charged leptons, neutrinos, c quarks and b quarks, that may be directly derived from the observables $\overline{s}_f$ and A_f ($f = l, \nu, Q$) are presented in Table 15, in comparison with SM predictions. The agrement with the SM is satisfactory for the charged leptons and c quarks, but the neutrino couplings show a 3.7σ, $\overline{v}_{\rm b}$ a 2.5σ and $\overline{a}_{\rm b}$ a 2.9σ deviation. As previously pointed out [16,17] the apparently anomalous behaviour of the b-quark couplings is essentially found in the right-handed effective coupling, $\overline{g}^R_{\rm b}$ rather than the left-handed one, $\overline{g}^L_{\rm b}$ where:

$$\overline{g}^R_{\rm b} = \frac{\overline{v}_{\rm b} - \overline{a}_{\rm b}}{2} \qquad (7.4)$$

$$\overline{g}^L_{\rm b} = \frac{\overline{v}_{\rm b} + \overline{a}_{\rm b}}{2} \qquad (7.5)$$

It is interesting to consider the history of this apparent anomaly, which is illustrated in Table 16. The most significant deviation (42% and 3.2σ) was seen in the 1996 data set [30]. In the current, essentially final, data set the size of the effect is reduced to 32% but still has a significance of 2.8σ. The left-handed coupling is now slightly more consistent with the SM (1.7σ deviation) as compared to 1998 (2.1σ deviation) and 1996 (1.8σ deviation). The experimental error on $\overline{g}^R_{\rm b}$ is reduced by $\simeq 40\%$ in the current data set as compared to that of 1996. The sources of these changes are made clear by the entries of Table 17 in which are presented the model-independent observables used to calculate the b-quark couplings, as derived from the 1996 and 2003 data sets. Also shown are the shifts of the observables in units of the 2003 experimental errors. It can be seen that the most important change occurs in the direct measurement of A_b from SLC. The value of A_l(LEP+SLC) used to extract A_b(LEP) from $A^{0,{\rm b}}_{\rm FB}$ is the same for the two data sets.

To date, only a few authors [42–44] have proposed new physics interpretations of the measured b-quark couplings. Other authors [13,40] have argued that there is unlikely to be a new physics interpretation of the observed anomaly. The present writer finds all the reasons given for this conclusion to be either simply wrong, or unconvincing. It was argued that: 'the sensitivity of $A^{0,{\rm b}}_{\rm FB}$ to A_b is small, because A_l is small'. In fact, the size of A_l is irrelevant. What are important are the *relative precisions* with which it and $A^{0,{\rm b}}_{\rm FB}$ are known. Since the errors on A_l and $A^{0,{\rm b}}_{\rm FB}$ are uncorrelated it follows from (3.1) that:

$$\frac{\sigma(A_b)}{A_b} = \sqrt{\left(\frac{\sigma(A_l)}{A_l}\right)^2 + \left(\frac{\sigma(A^{0,{\rm b}}_{\rm FB})}{A^{0,{\rm b}}_{\rm FB}}\right)^2} \qquad (7.6)$$

where $\sigma(X)$ is the experimental error on X. As shown in Tables 1 and 6, A_l and $A^{0,{\rm b}}_{\rm FB}$ have relative uncertainies of 1.2% and 1.6% respectively. Since the observed deviation from the SM is 3.6%, the contribution to the uncertainity on A_b due to that on A_l is essentially negligible in comparison with the the observed deviation from the SM prediction. It is further argued in [13] that a new physics interpretation is disfavoured because a significant deviation is seen only in $A^{0,{\rm b}}_{\rm FB}$ from LEP and not in R_b (equivalent to $\overline{s}_{\rm b}$) or A_b(SLC). But, in the case of a deviation from the SM only in the right-handed coupling, no significant change is expected in R_b[17]. Also the measured value of A_b(SLC) lies 1.5σ above A_b(LEP) and 0.5σ below the SM prediction. Furthermore $\chi^2_{A_b,WA}/d.o.f. = 2.32/1$, CL=0.13. There is therefore no strong evidence for any incosistency between the measured values of A_b(SLC) and A_b(LEP). Both values are included in the weighted average that differs by 2.5σ from the SM prediction. In [13] it is stated that: 'One concludes that most probably the observed discrepancy is due to a large statistical fluctuation and/or an experimental problem'. The discussion in Sect. 3 above reaches just the opposite conclusion. In fact the statement just quoted tacitly implies (without any justification) that all the LEP experiments have seriously underestimated the systematic arrors of their $A^{0,{\rm b}}_{\rm FB}$ measurements; indeed, as discussed in Sect. 3 above, by at least an order of magnitude. This may be possible, but hardly seems likely. The total estimated experimental error on the LEP value of $A^{0,{\rm b}}_{\rm FB}$ is is largely statistical. In contrast the R_b measurement has a statistical error of 0.20% to be compared with a systematic one of 0.22%, so that the estimated systematic contribution is much more important than that for $A^{0,{\rm b}}_{\rm FB}$. Indeed from general experimental considerations, asymmetries such as $A^{0,{\rm b}}_{\rm FB}$ are expected to have smaller systematic uncertainities than quantities such as R_b where absolute experimental detection efficencies play a role. Contrary to what is implied in [13] then, there is no objective reason to suppose that the $A^{0,{\rm b}}_{\rm FB}$ measurement should be less reliable than the R_b one. As discussed in Sect. 3, the hypothesis that the A_b deviation is a purely statistical effect has a CL of $\simeq 10^{-3}$ and so, though not completely excluded, is very unlikely. It remains true however that a correlated systematic error of unknown origin in the LEP $A^{0,{\rm b}}_{\rm FB}$ measurements is expected to produce an anomaly predominantly in the right-handed effective coupling. Assuming SM values for the couplings and that the -3.5% discrepancy in A_b is of systematic origin, the derived values of the couplings: $\overline{g}^R_{\rm b} = 0.09486$ and $\overline{g}^L_{\rm b} = -0.4187$ are in good agreement with the measured values in Table 16. Another argument [17] in favour of an unknown systematic origin for the A_b discrepancy is to note the good agreement of the measured value of $\overline{s}_{\rm b}$ with the SM prediction shown in Table 14. This agreement requires the presence of of large, m_t^2 dependent, quantum corrections originating in the strong breaking of quark flavour sym-

[16] These fitted values of m_H are for $\alpha(m_Z) = 0.007755$ and $m_t = 178$ GeV

[17] Since $\overline{s}_{\rm b} \simeq 2[(\overline{g}^R_{\rm b})^2 + (\overline{g}^L_{\rm b})^2]$ and, with the SM values of the couplings: $(\overline{g}^R_{\rm b})^2 = 0.005$, $(\overline{g}^L_{\rm b})^2 = 0.18$, a 100% deviation of $\overline{g}^R_{\rm b}$ from the SM prediction changes $\overline{s}_{\rm b}$ (or R_b) by only 8%.

Table 18. Constraints on the Z couplings to d-type quarks from the LEP average measurement of $\Gamma_{\rm had}$

$\Gamma_{\rm had}$(thy) definition	$\Gamma_{\rm had}$ [GeV]	$[\Gamma_{\rm had}$(thy)-$\Gamma_{\rm had}$(expt)]/ σ(expt)
SM	1.7439	−0.4
b → b(meas)	1.7451	0.35
b,d → b(meas)	1.7414	−1.50
b,d,s → b(meas)	1.7377	−3.4
$\Gamma_{\rm had}$(expt)	1.7444(20)	–

metry in the third generation of SM fermions. Neglect of these corrections gives a prediction of 0.3707 for $\overline{s}_{\rm b}$, differing from the measured value by 3.7σ. In the case of a new physics explanation of the A_b discrepancy, the appearence of the expected quantum corrections from the SM for $\overline{s}_{\rm b}$ must be regarded as fortuitous. Thus, although there are no objective experimental reasons to doubt the correctness of the A_b measurement, a systematic effect of unknown origin cannot be excluded. The good agreement of $\overline{s}_{\rm b}$ with the SM prediction and the large observed deviation in the right-handed coupling are consistent with this hypothesis. A purely statistical fluctuation is very unlikely. The effect could also be explained by new physics. There are no good reasons for the statement in [13] that: 'It is well known that this (A_b) discrepancy is not likely to be explained by some new physics effect in the $b\bar{b}$Z vertex'. In fact all the explanations mentioned above (including new physics) remain open possibilities. Only better experimental data can decide between them.

The next question that obviously arises is whether there is any evidence that the couplings of non-b quarks may also deviate from the SM predictions. Direct measurements of the light quark couplings have been performed by the DELPHI [45] and OPAL [46] Collaborations. For example, OPAL found:

$$\overline{g}^L_{\rm d,s} = -0.44^{+0.13}_{-0.09} \quad , \quad \overline{g}^R_{\rm d,s} = 0.13^{+0.15}_{-0.17}$$

in good agreement with the SM predictions of -0.424 and 0.077 respectively. However, the very large uncertainties on the measurements preclude obtaining any useful information concerning deviations at the few % level. such as that observed for the parameter A_b. In fact the observed deviations of A_b and A_c from the SM predictions by factors of 1.036 and 1.023 are of comparable size. Since the relative error on A_c is two times larger than that on A_b, only for the latter is a possibly significant deviation from the SM observed. The similar qualitative behaviour of A_b and A_c with respect to the SM prediction can be seen in Fig. 15.1 of [4]. The LEP measurement of $Q^{\rm had}_{\rm FB}$ can be used to extract an average value of $A_{\overline{q}}$ (averaged over all quark flavours) that is 1.07(7) times the SM prediction for this quantity. Thus, as previously mentioned, the different quark charge asymmetry measurements do not exclude deviations of A_q ($q = u, d, s, c$) as large as that observed for A_b. The present data are not, however, sufficiently precise to give any positive evidence for such an effect.

As pointed out in [47], much stronger constraints on the non-b quark couplings are provided by the LEP average value of the hadronic width, $\Gamma_{\rm had}$, of the Z boson [4]:

$$\Gamma_{\rm had} = 1.7444(20)\ {\rm GeV}$$

This value is in excellent agreement with the SM prediction[18] of 1.7439 GeV (0.03 %, 0.25σ deviation). One may also note in Table 14 the almost perfect agreement of the $\overline{s}'_{\rm nb}$ measurement with the SM prediction. The small relative uncertainty of 0.11% on $\Gamma_{\rm had}$ allows significant constraints to be placed on different hypotheses concerning the size of the Z$q\overline{q}$ couplings. Using the relation:

$$\Gamma_{\rm had} = \frac{\sqrt{2}G_\mu m_Z^3}{4\pi} \sum_q^{u,d,s,c,b} \overline{s}_q C_q^{\rm QED} C_q^{\rm QCD} \qquad (7.7)$$

the SM predictions for the d and s quarks may be replaced by the central values of the measured b-quark couplings from Table 14. The results given by replacing, in (7.7), the SM predictions for (i) b quarks, (ii) b and d quarks and (iii) b, d and s quarks by the measured b-quark couplings from Table 14 are presented in Table 18, in comparison with the measured value of $\Gamma_{\rm had}$. It can be seen that, although the prediction is little changed for case (i), case (iii) is excluded by the measured value of $\Gamma_{\rm had}$ at the 3.4σ level.

Since (see Table 14) the measured value of $\overline{s}_{\rm c}$ agrees well with the SM prediction, the measured value of $\Gamma_{\rm had}$ will provide no useful constraints if the procedure used in Table 18 is repeated for u-type quarks. The effective coupling constants $\overline{v}_{\rm c}$ and $\overline{a}_{\rm c}$ also agree well with the SM predictions.

The experimental situation concerning measurements of right-handed and left-handed Z-fermion pair couplings and the W boson mass is summarised in Tables 19 and 20. In Table 19 the couplings of charged leptons, c quarks, b quarks and neutrinos are compared with SM predictions. Similar comparisons are made in Table 20, varying the values of m_t and m_H in the SM predictions. In this case, for clarity, only deviations from the SM predictions are

Table 19. Measured values of precision electroweak parameters compared to SM predictions for $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_{\rm Z}) = 0.007755$

SM parameter	Expt value	SM	(Exp-SM)/σ
$\overline{g}^R_{\rm l}$	0.23171(25)	0.23202	−1.2
$\overline{g}^L_{\rm l}$	−0.26954(23)	−0.26935	−0.83
$\overline{g}^R_{\rm c}$	−0.1585(48)	−0.1547	−0.79
$\overline{g}^L_{\rm c}$	0.3460(36)	0.3468	−0.22
$\overline{g}^R_{\rm b}$	0.0951(63)	0.0774	2.8
$\overline{g}^L_{\rm b}$	−0.4182(16)	−0.4209	1.7
$\overline{g}^L_{\nu}$	0.5003(6)	0.50251	−3.7
m_W [GeV]	80.426(34)	80.394	0.94

[18] Unless otherwise stated, all SM predictions are for $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_{\rm Z}) = 0.007755$.

Table 20. Values of deviations (Expt-SM)/σ for precision electroweak parameters. SM predictions with $\alpha(m_Z) = 0.007755$

	$m_H = 120$ GeV			$m_H = 200$ GeV			$m_H = 300$ GeV		
m_t [GeV]	173.8	178.0	182.2	173.8	178.0	182.2	173.8	178.0	182.2
$\overline{g}_l^R$	−1.6	−1.2	−0.82	−2.4	−2.1	−1.7	−3.1	−2.8	−2.4
$\overline{g}_l^L$	−1.6	−0.83	0.03	−3.0	−2.1	−1.3	−4.0	−3.2	−2.4
$\overline{g}_c^R$	−0.77	−0.79	−0.80	−0.75	−0.76	−0.77	−0.72	−0.73	−0.75
$\overline{g}_c^L$	−0.19	−0.22	−0.27	−0.11	−0.16	−0.2	−0.06	−0.10	−0.15
$\overline{g}_b^R$	2.81	2.82	2.82	2.80	2.80	2.81	2.79	2.79	2.80
$\overline{g}_b^L$	1.71	1.70	1.64	1.60	1.57	1.54	1.52	1.49	1.46
$\overline{g}_\nu^L$	−3.5	−3.7	−3.9	−3.3	−3.5	−3.7	−3.2	−3.4	−3.6
m_W	1.7	0.94	0.18	2.6	1.9	1.1	3.4	2.7	1.9

Table 21. Quantum correction parameters for different fermion flavours f. SM predictions for $m_t = 178$ GeV, $m_H = 120$ GeV and $\alpha(m_Z) = 0.007755$

f	$\delta^f_{\rm Quant}$(Expt)	$\delta^f_{\rm Quant}$(Expt)/σ_f	$\delta^f_{\rm Quant}$(SM)	$\delta^f_{\rm Quant}$(SM)/σ_f	[Expt − SM]/σ_f
l	0.04064(92)	44.2	0.04118	44.8	−0.59
c	0.060(25)	2.4	0.0418	1.67	0.76
b	0.249(74)	3.4	0.049	0.65	2.7
ν	0.0006(12)	0.5	0.00502	4.2	−3.7

tabulated. Tables 19 and 20 contain, in concise form, essentially all the precision information on the SM derived from the experimental programmes of LEP and SLD as well as the main contributions of FERMILAB to the same subject (essentially measurements of m_t, m_W and the Z$\nu\overline{\nu}$ coupling) during the same period.

Looking at this comparison, it is difficult to conclude that the level of agreement with the SM is good. For $m_H = 120$ GeV, i.e. around the maximum value of $\overline{\rm CL}$, as in Table 19, $\overline{g}_b^R$ and $\overline{g}_\nu^L$ show deviations of 2.8 and −3.7 standard deviations respectively. All of $\overline{g}_l^R$, $\overline{g}_l^L$ and m_W show negative deviations around the one standard deviation level as a consequence of the low values of m_H (inconsistent with the experimental direct lower limit) favoured by the measured values of these quantities. For $m_H = 300$ GeV (see Table 20) five out of the eight EW parameters show deviations around three standard deviations. Those associated with $\overline{g}_b^R$ and $\overline{g}_\nu^L$ are almost independent of m_H and vary only weakly with m_t. Another feature is that increasing (decreasing) the value of m_t improves (worsens) the agreement for $\overline{g}_l^R$ and $\overline{g}_l^L$ (m_W) for all values of m_H. In any case, the SM still fares badly for $m_H = 300$ GeV and above. This is already apparent in Figs. 2 and 3, and will also be evident in the combined confidence level curves based on all precision data to be discussed below.

Perhaps the most important aspect of the SM that has been tested by recent precision measurements is the renormalisability of the theory. This enables quantum loop corrections involving fermions, weak bosons and the Higgs boson to be calculated and compared to experiment, just as precise measurements of similar effects in QED, such as the Lamb shift of Hydrogen and the anomalous magnetic moments of the electron and muon, were important to establish the essential correctness of the theory for the description of such quantities. The model-independent observables shown in Table 14 can be used to isolate the effect of quantum corrections in the coupling of charged leptons, c quarks, b quarks and neutrinos to the Z boson. For this only the values of A_l(lept), A_c and A_b are required for charged leptons and heavy quarks, and that of $\overline{s}_\nu$ for neutrinos. The A parameters are all simple mappings (see (3.2)) of the ratio: $r = v/a$ of the vector and axial-vector coupling constants. At tree-level in the SM the following relations hold:

$$r_l = 1 - 4s_W^2 \tag{7.8}$$

$$r_c = 1 - \frac{8}{3}s_W^2 \tag{7.9}$$

$$r_b = 1 - \frac{4}{3}s_W^2 \tag{7.10}$$

$$g_\nu^L = \frac{1}{2} \tag{7.11}$$

where $s_W^2 = \sin^2\theta_W^{\rm on-shell}$ as defined in (6.2). In the presence of quantum corrections $r_f \to \overline{r}_f$, and $s_W^2 \to (\overline{s}_W^2)^f$ ($f = l, c, b$) in (7.8)-(7.10) and $g_\nu^L \to \overline{g}_\nu^L$ in (7.11). Thus parameters, $\delta^f_{\rm Quant}$, that measure directly the effect of quantum corrections can be introduced according to the definitions:

$$\delta^l_{\rm Quant} \equiv \frac{(\overline{s}_W^2)^l - s_W^2}{s_W^2} = \frac{1-\overline{r}_l}{4s_W^2} - 1 \tag{7.12}$$

$$\delta^c_{\rm Quant} \equiv \frac{(\overline{s}_W^2)^c - s_W^2}{s_W^2} = \frac{3(1-\overline{r}_c)}{8s_W^2} - 1 \tag{7.13}$$

$$\delta^b_{\rm Quant} \equiv \frac{(\overline{s}^2_W)^b - s^2_W}{s^2_W} = \frac{3(1-\overline{r}_b)}{4s^2_W} - 1 \quad (7.14)$$

$$\delta^\nu_{\rm Quant} \equiv (\overline{g}^L_\nu - \frac{1}{2})/(\frac{1}{2}) = 2\overline{g}^L_\nu - 1 \quad (7.15)$$

In the absence of quantum corrections all the $\delta^f_{\rm Quant}$ parameters vanish. The experimental values of these parameters derived from the entries of Table 14, as well as the corresponding SM predictions, are presented in Table 21.

It can be seen from this table that the expected size of the corrections in the SM is $\simeq 4-5\%$ for l, c and b, and an order of magnitude lower for ν. By far the most significant measurement of quantum corrections (44 standard deviations from zero!) is that of $\delta^l_{\rm Quant}$. Good agreement with the SM prediction (1.2σ deviation) is found, at least for $m_H = 120$ GeV, as used in the SM predictions in Table 21. As discussed previously, in connection with $\overline{g}^R_{\rm l}$ and $\overline{g}^L_{\rm l}$ (see Table 20), the agreement worsens considerably for higher values of m_H. For example, for $m_H = 300$ GeV, $\delta^l_{\rm Quant}({\rm SM}) = 0.04385$, a deviation of 3.5σ from the measured value. For c quarks the expected quantum correction is only a 1.7σ effect. Good agreement with the SM prediction is found in this case. For b quarks the expected quantum correction is unmeasurable with present data (expected effect 0.64σ from zero) whereas a 3.4σ effect is actually observed, differing by 2.7σ from the SM prediction. This is indicative, as discussed previously, of either a large and unknown correlated systematic effect (perhaps in combination with a statistical fluctuation) in the LEP $A^{0,{\rm b}}_{\rm FB}$ measurements, or of new physics at tree-level, or, indeed some combination of the two. A purely statistical fluctuation is very unlikely. The situation is exactly the opposite for the neutrino couplings. The SM predicts a large (4.2σ) quantum correction, while in the data only a 0.5σ effect is seen. The discrepancy with the SM amounts to -3.7σ in this case. Rather than a tree-level effect giving a large apparent quantum correction, as for the b quarks, it seems that the SM quantum corrections are effectively 'turned off' in the case of the the $Z\nu\overline{\nu}$ couplings! Theoretical interpretations of this apparent coupling suppression have been made in [48,49].

The main conclusion to be drawn from the results presented in Table 21 is that only for the charged leptons and c quarks is the present data reasonably consistent with the SM. This implies that since the c-quark couplings are almost completely insensitive to the values of m_t and m_H (see Table 5) information on these parameters, via quantum corrections, can only be reliably obtained from the charged lepton couplings. This further implies (see Tables 11 and 12) that the maximum values of $\overline{\rm CL}$ of 23% or 1.7% are obtained at $m_H = 115$ GeV when the NuTeV m_W measurement is excluded or included respectively. Much lower confidence levels of 0.054 and 0.0011 are obtained, under the same conditions, for $m_H = 220$ GeV, the presently quoted 95% CL upper limit on m_H from the EWWG [4]. In fact, even lower CLs will be found for a global analysis based on all model-independent observables shown in Table 14, which will now be discussed.

Table 22. Combined confidence levels $\overline{CL}$ for consistency with the SM as a function of m_H. All data, as in Fig. 6

m_H (GeV)	NuTeV out	NuTeV $\overline{s}_\nu$ meas.	NuTeV m_W meas.
111	4.9×10^{-7}	4.9×10^{-8}	3.2×10^{-8}
113	2.0×10^{-4}	2.1×10^{-5}	1.4×10^{-5}
115	0.031	3.8×10^{-3}	2.6×10^{-3}
140	0.051	5.1×10^{-3}	4.2×10^{-3}
180	0.053	5.9×10^{-3}	6.4×10^{-3}
220	0.041	5.6×10^{-3}	6.4×10^{-3}
260	0.026	2.9×10^{-3}	5.0×10^{-3}
300	0.015	1.6×10^{-3}	3.4×10^{-3}

Table 23. Combined confidence levels $\overline{CL}$ for consistency with the SM as a function of m_H. Lepton data only, as in Fig. 7

m_H (GeV)	NuTeV out	NuTeV $\overline{s}_\nu$ meas.	NuTeV m_W meas.
111	1.3×10^{-6}	7.4×10^{-8}	4.7×10^{-8}
113	4.5×10^{-4}	2.8×10^{-5}	1.9×10^{-5}
115	0.061	4.7×10^{-3}	3.3×10^{-3}
140	0.042	2.2×10^{-3}	2.0×10^{-3}
180	0.010	5.3×10^{-4}	7.4×10^{-4}
220	2.0×10^{-3}	1.0×10^{-4}	2.1×10^{-4}
260	3.5×10^{-4}	1.9×10^{-5}	5.3×10^{-5}
300	6.0×10^{-5}	3.4×10^{-6}	1.2×10^{-5}

In order to test the overall level of agreement of the precision data with the SM, χ^2 estimators are calculated using the observables shown in Table 14. The corresponding confidence level ${\rm CL}(\chi^2_{\rm SM})$ is then combined with that, ${\rm CL}_{s+b}$, of the direct Higgs search using (2.8) to give m_H confidence level curves, $\overline{\rm CL}$. Since all equivalent and statisically compatible measurements are combined in order to extract the model-independent observables, there is, in this case, no contribution of the type $\chi^2_{X,WA}$ to take into account the degree of statistical compatiblity of different measurements of the same quantity X. In view of the possibility of new physics or poorly understood systematic effects for the $Zb\overline{b}$ couplings, and of the overall status, as well as the different possible interpretations of the NuTeV experiment, the above procedure is repeated for different selections and interpretations of the data: the results are presented in Tables 22 and 23 and Figs. 6 and 7.

In Table 22 and Fig. 6, all the observables in Table 14, except $\overline{s}_{\rm c}$, are included in the χ^2 estimator. In the case that the NuTeV result is excluded, $\overline{s}_\nu$ is assigned the LEP-only value of Table 5 and (6.7). The 'NuTeV $\overline{s}_\nu$ meas.' curves use the $\overline{s}_\nu$ value quoted in Table 14 and (6.9) (LEP+NuTeV average). For the interpretation of the NuTeV re-

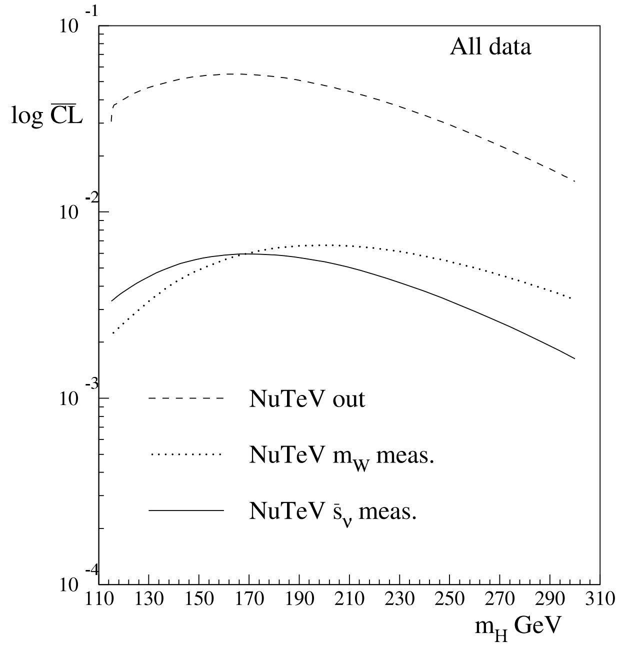

Fig. 6. Combined m_H confidence levels. CL_{s+b} is combined with $\mathrm{CL}(\chi^2_{\mathrm{SM}})$ using (2.8). χ^2_{SM} is calculated from all model-independent variables: A_l(lept), $\overline{s}_l$, A_c, A_b, $\overline{s}_{\mathrm{b}}$, $\overline{s}'_{\mathrm{nb}}$, $\overline{s}_\nu$, m_W and m_W(NuTeV)

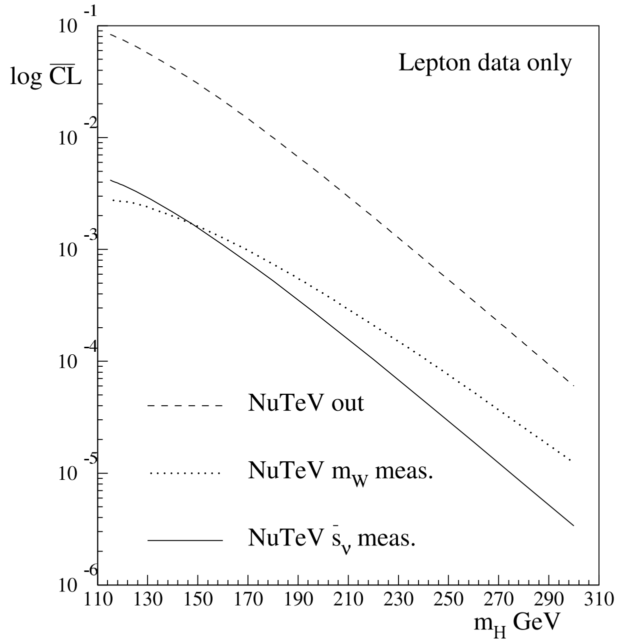

Fig. 7. Combined m_H confidence levels. CL_{s+b} is combined with $\mathrm{CL}(\chi^2_{\mathrm{SM}})$ using (2.8). χ^2_{SM} is calculated from the leptonic model-independent variables: A_l(lept), $\overline{s}_l$ $\overline{s}_\nu$, as well as m_W and m_W(NuTeV)

sult as an m_W measurement, m_W(NuTeV) is included in χ^2 and $\overline{s}_\nu$ is set to the LEP-only value. The number of degrees of freedom of the 'NuTeV out','NuTeV $\overline{s}_\nu$ meas.' and 'NuTEV m_W meas.' estimators are 10, 10 and 11 respectively.

In the case of new physics, or unknown systematic effects, in the A_b measurement, it is clearly of interest to test only the level of agreement of the leptonic sector with the SM. For this, combined m_H confidence level curves are also derived using only the leptonic observables: A_l(lept), $\overline{s}_l$ and $\overline{s}_\nu$ as well as m_W and, possibly m_W(NuTeV). The 'NuTeV out','NuTeV $\overline{s}_\nu$ meas.' and 'NuTeV m_W meas.' cases are considered as previously, giving now 4, 4 and 5 degrees of freedom, respectively, for χ^2_{SM}. The results for the 'Lepton data only' case are presented in Table 23 and Fig. 7.

It can be seen from Table 22 and Fig. 6 that, when all data is included, the maximum of $\overline{\mathrm{CL}}$ occurs around $m_H = 180$ GeV, and has a value $\simeq 0.05$ in the case that the NuTeV experiment is excluded and $\simeq 0.006$ when it is included, independently of the interpretation ($\overline{s}_\nu$ or m_W measurement) of the experiment. For $m_H = 300$ GeV, the CLs are much lower, being 0.015, 0.0016 and 0.0034 for the 'NuTeV out','NuTeV $\overline{s}_\nu$ meas.' and 'NuTeV m_W meas.' cases respectively. In the case that only the leptonic Z-decay data is considered, the maximum value of $\overline{\mathrm{CL}}$ occurs just above the direct lower limit with the values $\simeq 0.06$ (NuTeV out) and $\simeq 0.004$ (NuTeV included). For $m_H =$ 300 GeV, very low values of $\overline{\mathrm{CL}}$ are found: 6.0×10^{-5}, 3.4×10^{-6} and 1.2×10^{-5} for the three cases considered previously.

8 Comparison with the EWWG and EWPDG global fits

The conclusions presented above concerning the global level of agreement with the SM, and the possible value of m_H may seem somewhat at variance with those drawn from the global fits of the EWWG [4] and EWPDG [5]. Indeed, the quoted χ^2 confidence levels of the latter are much higher. The EWWG quotes confidence levels of 4.5% for 'all data' and 28% when NuTeV is excluded. These fits obtain (see Table 2) $m_H = 96^{+60}_{-35}$ GeV and $m_H = 91^{+55}_{-36}$ GeV respectively. Although the 1σ confidence bands contain regions allowed by the direct experimental lower limit on m_H, the central fit values are excluded by this limit. Already for $m_H = 111$ GeV, $\mathrm{CL}_{s+b} = 10^{-6}$ and for $m_H = 96$ GeV, $\mathrm{CL}_{s+b} \ll 10^{-8}$ (see Fig. 1). In addition the fitted values of m_H are strongly biased towards higher values, as discussed above, due to the inclusion of the 'anomalous' quark asymmetry data in the fits. The latest global EWPDG fit, which did not include the NuTeV datum, found $m_H = 98^{+51}_{-35}$ GeV and $\chi^2/d.o.f. = 47.3/38$, CL= 14%. Some reasons for the higher confidence levels found for the global EWWG and EWPDG fits have been put forward in Sect. 2 above, so it is interesting, in the light of that discussion, to examine in detail the contributions of different types of observables to the χ^2 of these fits.

The 20 observables included in the EWWG fit may be classified as follows:

Measured: $\Delta^{(5)}_{\mathrm{had}}(m^2_Z)$, m_Z, m_t.
m_H-sensitive: $A^{0,l}_{\mathrm{FB}}$, $A_l(P_\tau)$, $\sin^2\Theta^{\mathrm{lept}}_{\mathrm{eff}}(Q^{\mathrm{had}}_{\mathrm{FB}})$, $A_l(SLD)$, $A^{0,\mathrm{b}}_{\mathrm{FB}}$, $A^{0,\mathrm{c}}_{\mathrm{FB}}$, m_W, $\sin^2\Theta_W(\nu\mathcal{N})$.

Others: σ_h^0, $\Gamma_{\rm Z}$, R_l^0, R_b^0, R_c^0, A_b, A_c Γ_W, $Q_W(Cs)$.

The relative contributions to the total χ^2 of the different types of observable are summarised in the first three rows of Table 24. The quantities $\sum \Delta(\chi')^2$ are calculated from the squares of the 'pulls' given in the last column of Table 16.1 of [4]. Also shown in Table 18 are the 'pseudo-confidence levels', CL', for each $\sum \Delta(\chi')^2$ assuming that the effective number of degrees of freedom, $d.o.f.'$, is equal to the number of observables of each type. Since there are five fitted parameters: m_Z, m_t, $\log(m_H)$, $\alpha(m_{\rm Z})$ and $\alpha_s(m_{\rm Z})$, the true total number of degrees of freedom is 15 rather than 20, so that the quoted values of CL' should be considered as upper limits on the true CL to be associated with each type of observable. This procedure neglects correlations between the fitted parameters, but is adequate to show the very different contributions of the different observable types to the overall χ^2. The 'Measured' observables, i.e. those that are identical to fitted parameters, are seen to provide an essentially vanishing contribution to χ^2, since the fitted parameters are completely determined by the corresponding observables. Thus three extra degrees of freedom are obtained, free of charge, which considerably improve the CL of the SM comparison[19]. The Measured parameters should be fixed in the fit, not treated as observables to be fitted. The effective confidence level, CL' of the 8 'm_H-sensitive' observables is a factor of 21 smaller than the CL' of all observables (see Table 24) and a factor 6.4 smaller than the EWWG global fit CL. The value of CL' for the m_H-sensitive observables is similar to the maximum value 0.0064 of $\overline{\rm CL}$ for the analysis of all model-independent observables in the last column of Table 22 and Fig. 6. The 'Other' observables give a rather low contribution to the total $(\chi')^2$, as compared to the expectation from the number of degrees of freedom, which give a further improvement to the CL of the global fit beyond that expected from the inclusion of observables that are only weakly sensitive to m_t and m_H. Excluding the NuTeV datum $\sin^2\Theta_W(\nu\mathcal{N})$ from the group of m_H-sensitive observables gives $\sum \Delta(\chi')^2 = 12.6$, $d.o.f' = 7$, CL'$= 0.08$ which is similar to the maximum value of $\overline{\rm CL}$ of 0.053 for the corresponding analysis of model-independent observables in Fig. 6 and the first column of Table 22[20] Thus the difference between the confidence levels found in the present paper and those of the global EWWG fits can be largely understood as a consequence of the extra degrees of freedom associated with the 'Measured' and 'Other' observables which are only weakly sensitive to the crucial unknown parameter, m_H, of the SM.

Classifying the 42 observables used in the latest global EWPDG fit [5] in the same fashion as done above for the EWWG fit gives:

Measured: m_Z, m_t.
m_H-sensitive: $A_{\rm FB}^{0,e}$, $A_{\rm FB}^{0,\mu}$, $A_{\rm FB}^{0,\tau}$, $A_{\rm FB}^{0,{\rm b}}$, $A_{\rm FB}^{0,{\rm c}}$, $A_{\rm FB}^{0,s}$, $\sin^2\Theta_{\rm eff}^{\rm lept}(Q_{\rm FB}^{\rm had})$, $A_e(1)$, $A_e(2)$, $A_e(3)$, A_μ, $A_\tau(1)$, $A_\tau(2)$, m_W(FERMILAB), m_W(LEP)
Others: $\Gamma_{\rm Z}$, $\Gamma_{\rm had}$, $\Gamma_{\rm inv}$, $\Gamma_{l^+l^-}$, σ_h^0, R_e^0, R_μ^0, R_τ^0, R_b^0, R_c^0, A_b, A_c, A_s, R^-, κ^ν, $R^\nu(1)$, $R^\nu(2)$, $g_V^{\nu e}(1)$, $g_V^{\nu e}(2)$, $g_A^{\nu e}(1)$, $g_A^{\nu e}(2)$, $Q_W(Cs)$, $Q_W(Tl)$, $\Gamma(b \to s\gamma)/\Gamma(c \to e\nu)$, $(g_\mu - 2 - \alpha/\pi)/2$

The difference with respect to the EWWG fit is that a wider range of observables are included as well as several different measurements of the same quantity, indicated, for example,as $A_e(1)$,$A_e(2)$..., and that charged lepton universality has not been used to reduce the number of observables. It could be argued that some of the 'Other' observables such as $\Gamma_{\rm Z}$, $\Gamma_{\rm had}$ and $\Gamma_{\rm inv}$ are actually quite 'm_H-sensitive', but the choice of the latter type of observable has been restricted, for purposes of comparison, to correspond as closely as possible to that made above for the EWWG fit. The values of $\sum \Delta(\chi')^2$, $d.o.f.'$ and CL' for the three classes of observables in the EWPDG fit are presented in the fifth, sixth and seventh rows of Table 24. There are only two Measured observables, m_t and m_Z, since the fitted parameters are: m_t, m_Z, m_H and $\alpha(m_{\rm Z})$. Unlike for the EWWG fit $\alpha_s(m_{\rm Z})$ is treated as a fixed rather than a fitted parameter. The high value of CL' for the Measured observables shows that, as in the case of the EWWG fit, it is more appropriate to treat m_t and m_Z as fixed parameters in the fit. As for the EWWG fit, the value of CL' for the m_H-sensitive observables is much less than the global fit confidence level. The value of CL' for the 'Other' observables gives no indication for a possible over-estimation of systematic errors as in the EWWG fit. However, the observable $(g_\mu - 2 - \alpha/\pi)/2$, not included in the EWWG fit, shows a quite large deviation from the SM prediction. Removing this observable from the 'Others' set gives: $\sum \Delta(\chi')^2 = 17.8$, $d.o.f.' = 24$, CL'$= 0.81$., similar to the value 0.78 found in the EWWG fit.

Thus the lower confidence levels found in the global analysis of the present paper as compared to those quoted by the EWWG and EWPDG are fully explained in terms of the dilution of the hypothesis testing power of the latter fits due to the inclusion of unaveraged or insensitive observables in the χ^2 estimator.

It is interesting to compare the EWWG and EWPDG fits to the the essentiallly final LEP+SLD data set discussed above to an earlier comparison of Hagiwara *et al.* [50] based on the 1996 data set. The observables used were:

Measured: none
m_H-sensitive: $A_{\rm FB}^{0,l}$, A_τ, A_e, $A_{\rm FB}^{0,{\rm b}}$, $A_{\rm FB}^{0,{\rm c}}$, $\sin^2\Theta_{\rm eff}^{\rm lept}(Q_{\rm FB}^{\rm had})$, $A_l(SLD)$, m_W(FERMILAB).
Others: σ_h^0, $\Gamma_{\rm Z}$, R_b^0, R_c^0, A_b, A_c, K(CCFR).

[19] For example, calculating CL' by summing the values of $\sum \Delta(\chi')^2$ corresponding to each observable type gives $(\chi')^2/d.o.f.' = 26.64/20$, CL'$= 0.15$ when the Measured observables are included (see Table 24) and $(\chi')^2/d.o.f.' = 26.60/17$, CL'$= 0.064$ when they are excluded.

[20] Actually the EWWG and EWPDG fits use the old m_t value 174.3(5.2) GeV rather than 178.0(4.3) as used in Tables 22 and 23. This, however, has only a minor effect on the maximum values of $\overline{\rm CL}$. Using the old value of m_t, maximum values of $\overline{\rm CL}$ of 0.0080 [0.057] are found when the NuTeV experiment is included [excluded], to be compared with the corresponding numbers 0.0064 [0.053] quoted above using the new m_t measurement.

Table 24. Contributions of different types of observables to the χ^2 of the latest global EWWG [4] and EWPDG [5] fits and that of Hagiwara et al. [50]. See text for definitions of the quantities shown

	Observable type	$\sum \Delta(\chi')^2$	$d.o.f.'$	CL'
	Measured	0.04	3	0.998
EWWG	m_H-sensitive	21.0	8	0.0071
	Others	5.6	9	0.78
	All	26.6	20	0.15
	Measured	0.05	2	0.975
EWPDG	m_H-sensitive	25.0	15	0.050
	Others	25.1	25	0.46
	All	50.2	42	0.18
	Measured	–	–	–
Hagiwara	m_H-sensitive	14.2	8	0.08
et al.	Others	9.2	7	0.24
	All	23.4	16	0.10

The corresponding pseudo-χ^2 values, that are presented in the last three rows of Table 24, correspond to the parameter settings: $m_t = 175$ GeV, $m_H = 100$ GeV, $\alpha(m_Z) = 0.007767$ and $\alpha_s(m_Z) = 0.118$. As in the current data sets the largest contribution to $\sum \Delta(\chi')^2$ comes from the m_H-sensitive observables. In this case however this is due essentially to the single observable $A_{\rm FB}^{0,{\rm b}}$. Removing it gives $\sum \Delta(\chi')^2 = 6.9$, $d.o.f.' = 7$, $CL = 0.43$. Similar removal of $A_{\rm FB}^{0,{\rm b}}$ from the recent EWWG data set above gives $\sum \Delta(\chi')^2 = 15.2$, $d.o.f.' = 7$, $CL = 0.033$. The large remaining $\sum \Delta(\chi')^2$ value is due to the NuTeV datum. As previously remarked in Sect. 3 above, the significance of the $A_{\rm FB}^{0,{\rm b}}$ deviation has been stable, since 1996, at the 2.5σ level, in spite of a 30% reduction in the total error on this quantity.

A final remark concerning the CLs quoted for the EWWG and EWPDG fits is that, as previously mentioned, these numbers correspond to central fitted values of m_H that are lower than the direct experimental lower limit of 114.4 GeV. Replacing the fitted values by this limit will evidently yield lower CLs. Because of the relatively large uncertainty on the fitted value of m_H, the expected reduction in the CL will be less than that due to correcting for the statistical dilution effects discussed above.

The latest EWWG report [4], quotes a 95 % confidence level upper limit on m_H of 219 GeV. This estimate is based on the famous 'blue band'[21] plot (Fig. 16.5 of [4]) which shows the quantity: $\Delta\chi^2 = \chi^2 - \chi^2_{\rm min}$ for the global fit as a function of m_H. Choosing an appropriate fixed value of $\Delta\chi^2$ to define a confidence limit for m_H is a valid procedure only in the case that the confidence level derived from the value of χ^2 of the fit to the m_H-sensitive variables is sufficiently high. Inspection of Figs. 2 and 3 and Tables 11 and 12 shows that this is hardly the case for the actual electroweak data set. It was argued in the previous section that most reliable estimate of m_H is that derived using only charged lepton asymmetry and polarisation data and m_W. In this case the the maximum value of $\overline{\rm CL}$ is 0.017 (see Table 12) when the NuTev m_W measurement is included, 0.23 (see Table 11), if it is excluded. Thus the $\Delta\chi^2$ estimator for m_H is acceptable only for the case of a fit to A_l(lept) and m_W excluding the NuTeV measurement. However, at the 95 % confidence level EWWG upper limit of $m_H = 220$ GeV, the corresponding values of $\overline{\rm CL}$ are 1.1×10^{-3} (NuTeV in) and 0.01 (NuTeV out) with the implication that if a Higgs boson existed with this mass, and including, as in the final EWWG fit, the NuTeV measurement, the SM would be excluded by the data with a confidence level only slightly larger than 10^{-3}! This is not at all the message that one might naively draw from the corresponding EWWG numbers. A confidence level of 4.5% for a global fit gives the impression that the data is, in general, not too badly described by the SM. In fact as a brief inspection of Tables 19-23, which contain all relevant information, show, this is hardly the case, so that a meaningful estimate of m_H cannot be derived from a $\Delta\chi^2$ plot based on such a global fit. In the model-independent analysis of all precision data in the previous section, the level of the discrepancy with the SM may be even larger than if only the m_H-sensitive observables A_l and m_W are considered. Referring to Figs. 6 and 7 and Tables 22 and 23 and taking the statistically favoured ' NuTeV $\overline{s}_\nu$ meas.' curves, gives values of $\overline{\rm CL}$ of 0.0038 (0.0056) for $m_H = 115$ (220) GeV for the 'All data' case and 0.0047 (1.0×10^{-4}) for the most reliable 'Lepton only' data. The reader must judge for herself (or himself) whether, in these circumstances, the SM does, or does not, provide an adequate description of the current data. Only in the case that it does, the 'blue band' plot, derived from a global fit to this data, provides a meaningful upper limit on m_H.

[21] So-called becuse of the coloured version shown in dozens of electroweak review talks over the last decade

9 Summary and conclusions

It has been demonstrated in this paper that the the statistical estimator, $\chi^2_{\rm data,SM}$ universally employed by the EWWG and EWPDG to judge the level of overall agreement of precision electroweak data with the SM predictions typically yields a confidence level an order of magnitude higher than estimators chosen to test specifically this level of agreement rather than the internal consistency of different measurements. The reasons for this are discussed in Sect. 2. While, as shown in Sect. 4, the number of independent observables sensitive to the most important poorly known parameters m_t and m_H is very small, large numbers of observables (20 for EWWG, 42 for EWPDG) are used to construct $\chi^2_{\rm data,SM}$. In these circumstances the corresponding CL reflects more the internal consistency of measurements of different observables than the level of agreement of the essential 'refined' parameters with the SM. Further dilution of the hypothesis testing power of $\chi^2_{\rm data,SM}$ results from the inclusion of many observables only weakly sensitive to m_t and m_H as well as fit parameters identical to measured quantities that give anomalously low contributions to the global χ^2.

In Sect. 3 the internal consistency of different heavy quark asymmetry measurements is discussed in detail. It is found to be very good. Combination of the statistically independent χ^2 and Run Tests for the LEP and SLD measurements of A_b yields a CL of 2.5×10^{-3} for a purely statistical fluctuation, which therefore seems unlikely. Attributing the A_b deviation to a correlated systematic error of unknown origin in the LEP $A_{\mathrm{FB}}^{0,\mathrm{b}}$ measurements would require an effect that is 1.8 times larger than the estimated QCD correction, which is the dominant source of correlated systematic uncertainty on this quantity, and 13 times larger than the estimated uncertainty on this correction. Even given the inevitable theoretical uncertainties associated with QCD effects this again seems unlikely. Thus there is no sound experimental reason to doubt, as suggested by some authors [13,40], the validity of the b-quark asymmetry measurements and the possible evidence they provide for new physics beyond the SM.

It is shown in Sect. 4 that essentially all information from quantum loop effects on the values of m_t and m_H is provided by only three observables: A_l(lept), $\overline{s}_l$ and m_W (see Table 5). For m_H there are only two strongly sensitive observables: A_l(lept) and m_W. It is demonstrated by fitting only the m_H-sensitive observables and also complete sets of model-independent observables that indeed the fitted value of m_H is essentially determined by the former set only. In an approach similar to that followed in [19] it is assumed, in Sect. 4, that the SM is valid so that different values of A_l may be extracted from purely leptonic data (A_l(lept)) and from quark forward/backward asymmetries (A_l(had)). As previously noted [19], for the case of the equivalent observable $\sin^2 \Theta_{\mathrm{eff}}^{\mathrm{lept}}$, these two estimates differ by 3.0σ showing that the SM interpretation of the data is also inconsistent at this level. This is a simple consequence of the observed anomalous behaviour of A_b and the possibly similar behaviour of other $\mathrm{Zq\overline{q}}$ couplings. Also, as previously noticed, much larger values of m_H are favoured by A_l(had) than by A_l(lept). This is because (see Table 5) in the SM only A_l, not A_b, is sensitive to m_H and the measured forward backward asmmetry $A_{\mathrm{FB}}^{0,\mathrm{b}}$ is, as shown in (3.1), proportional to $A_l A_b$. Whatever the explanation of the A_b anomaly, the most reliable estimate of m_H that can be derived from the present data is therefore that provided by A_l(lept). Inclusion of A_l(had), assuming, in contradiction with the observed A_b value, the correctness of the SM, gives a calculable bias towards higher values of m_H resulting from the A_l-A_b correlation in the quark forward/backward asymmetry.

In Sect. 5 a combined confidence level, $\overline{\mathrm{CL}}$, as a function of m_H is derived from the directly measured [10] confidence level curve CL_{s+b} and the CL curve derived from the χ^2 estimator using the m_H-sensitive observables A_l and m_W (see Fig. 1). As in Sect. 4 the correctness of the SM is assumed and different $\overline{\mathrm{CL}}$ curves are calculated for A_l(lept), A_l(had) and A_l(all), where the latter is the weighted average of the former two observables. The above mentioned inconsistency between A_l(lept) and A_l(had) is taken into account when calculating the $\overline{\mathrm{CL}}$ curve for A_l(all). Consistent results for $\overline{\mathrm{CL}}$ (see Tables 9 and 10) are found using either $\chi^2_{\mathrm{data,WA}} + \chi^2_{\mathrm{WA,SM}}$ directly or combining the CLs of $\chi^2_{\mathrm{data,WA}}$ and $\chi^2_{\mathrm{WA,SM}}$ using (2.8). Only the standard interpretation of the NuTeV experiment, as a measurement of m_W, is considered in Sect. 4, and the $\overline{\mathrm{CL}}$ curves are calculated both including and excluding this datum. The results for $\overline{\mathrm{CL}}$ are shown in Tables 9-12 and Figs. 2 and 3. The inconsistency of the SM interpretation is evident on inspection of these figures. For $m_H = 300$ GeV values of $\overline{\mathrm{CL}}$ differing by two orders of magnitude are obtained from A_l(lept) and A_l(had). Also studied in Sect. 5 is the dependence of the $\overline{\mathrm{CL}}$ curves on the assumed values of m_t and $\alpha(m_{\mathrm{Z}})$ (Table 13 and Figs. 4 and 5). Variation of m_t by plus or minus the experimental uncertainty changes the value of $\overline{\mathrm{CL}}$ by more than three orders of magnitude for $m_H \simeq 300$ GeV; somewhat smaller changes are given by a similar variation of $\alpha(m_{\mathrm{Z}})$. This demonstrates the importance of more precise measurements of these parameters in order to obtain well defined SM predictions.

In Sect. 6 the alternative interpretation of the result of the NuTeV experiment as a measurement of the $\mathrm{Z}\nu\overline{\nu}$ coupling, rather than m_W, is considered. It is pointed out that the former interpretation (in an analysis where is is assumed, consistent with the measured values of Γ_{had}, $\overline{s}'_{\mathrm{nb}}$ and $\overline{s}_{\mathrm{c}}$, that the $\mathrm{Z}q\overline{q}$, $q = \mathrm{u,d}$ couplings agree with SM predictions) is highly favoured by a statistical argument based on the internal consistency of LEP and NuTeV data. In this case the model-independent observable $\overline{s}_\nu$ derived from the LEP and NuTeV data differs from the SM prediction by 3.7σ and so is the largest single deviation observed from a SM prediction. The combined CL, taking into account both agreement with the SM and data consistency, is very similar for either interpretation of the NuTeV result.

An analysis in terms of model-independent observables similar to those previously published for earlier electroweak data sets [16–18,47] is presented in Sect. 7. Results are presented in terms of the ‘maximally uncorrelated’ observables presented in Table 14. Vector and axial vector couplings of the Z to fermion pairs (Table 15), and the equivalent right-handed and left-handed couplings (Table 19) are also presented and compared with SM predictions. The history of, and the different possible physical interpretations of, the A_b anomaly are also discussed. Although there are no purely experimental reasons for doubting the correctness of the fully compatible LEP and SLD measurements of A_b it is pointed out that the good agreement between the measured values of $\overline{s}_{\mathrm{b}}$ and the SM prediction (which requires the presence of large m_t dependent quantum corrections) must be fortuitous if the A_b anomaly is to be explained by new physics. This is an argument suggesting an unknown systematic bias as the cause of the effect. It is also pointed out that such a systematic bias would result in an anomaly predominantly in the right handed coupling (2.8 σ effect observed) rather than in the left-handed one (1.7 σ effect observed). This implies that only new, more precise, experiments can discriminate between new physics and unknown systematic bias as the

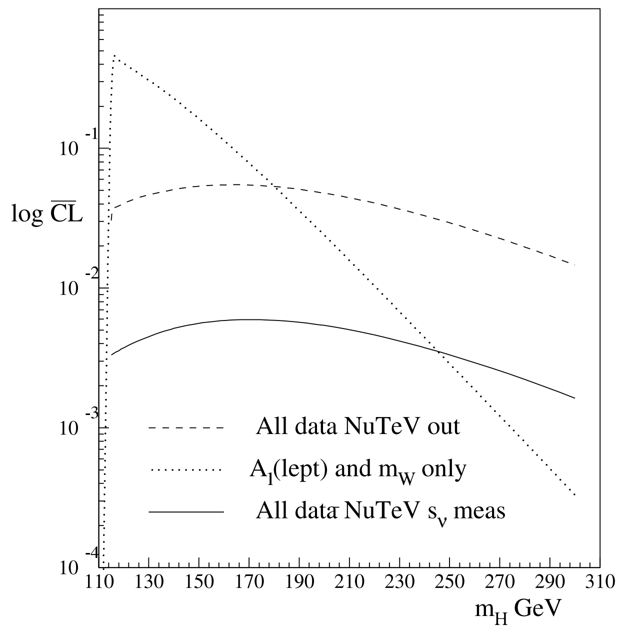

Fig. 8. Combined m_H confidence levels. See Fig. 6 for the definition of 'All data'

cause of the observed anomaly in the right handed b-quark coupling. It is shown that the measured value of Γ_{had} renders unlikely the possiblity that the couplings of the other d-type quarks differ from the SM prediction in the same way as observed for the b-quark couplings. The existing direct measurements of these couplings are insufficiently precise to provide any useful constraints.

In Sect. 7 the observed and expected quantum loop corrections are also discussed (see Table 21). The most significant and the most precisely measured effect, $\delta^l_{\mathrm{Quant}}$, defined in (7.12) is a 4% effect measured with a relative precision of 2.2% (measured effect forty-four standard deviations from zero). For a value of m_H of 120 GeV, the agreement of $\delta^l_{\mathrm{Quant}}$ with the SM prediction is quite satisfactory (1.2σ deviation) but, as shown in Table 20, for higher values of m_H the level of agreement of the charged lepton couplings that determine $\delta^l_{\mathrm{Quant}}$ deteriorates rapidly. $\delta^c_{\mathrm{Quant}}$ (7.13) is also predicted to be $\simeq 0.04$ in good agreeement with the measurement (0.7σ deviation) but here the fractional precision of the measurement is only 42%. $\delta^b_{\mathrm{Quant}}$ (7.14) is predicted to be $\simeq 0.05$ with an expected relative accuracy of 151%, so that no significant measurement is to be expected in this case. In fact the measured value of $\delta^b_{\mathrm{Quant}}$ is much larger, 0.249(74), and so requires new physics at the tree level (2.7σ deviation from the SM) if the data is correct. In contrast the experimental value of $\delta^\nu_{\mathrm{Quant}}$ (7.15) is 0.00502 with an expected relative experimental uncertainty of 24% (4.2σ deviation from zero) whereas the measured value is 0.0006(12) (0.5σ deviation from zero). In this case the expected quantum corrections are not observed, leading to a -3.7σ deviation from SM prediction. Some theoretical interpretations of this effect have already been proposed [48, 49].

Finally, in Sect. 7, combined confidence level curves $\overline{\mathrm{CL}}$ are derived including in the χ^2 estimator not only the m_H-sensitive observables, as in Sect. 5, but all, or chosen subsets of, other model-independent observables. In order to see the impact of the NuTeV experiment, $\overline{\mathrm{CL}}$ curves are presented excluding the NuTeV data or for the two alternative interpretations: $\overline{s}_\nu$ or m_W measurements. The same set of $\overline{\mathrm{CL}}$ curves is also obtained using only the leptonic observables: A_l(lept), $\overline{s}_l$ and $\overline{s}_\nu$ in addition to m_W. The results are shown in Tables 6 and 7 and Tables 22 and 23. The curves where NuTeV is included lie about an order of magnitude below those where it is excluded. The $\overline{\mathrm{CL}}$ curves for the two different interpretations of the NuTeV experiment are similar, except that those corresponding to an $\overline{s}_\nu$ measurement lie significantly lower for large values of m_H of ≥ 300 GeV.

In Sect. 8 a comparison is made between the CLs found previously in the present paper and those quoted for the latest published EWWG and EWPDG global fits, as well as an earlier fit by Hagiwara *et al.* [50]. They are shown to be quite consistent when the various dilution effects of the χ^2 estimators used in the global fits are taken into account. The maximum value of $\overline{\mathrm{CL}}$ using all data and choosing the $\overline{s}_\nu$ measurement interpretation of the NuTeV experiment of 0.0059, at $m_H = 180$ GeV, is about an order of magnitude lower than the CL of 4.5% quoted for the 'all data' EWWG fit. The CLs of the global EWWG and EWPDG fits correspond to central fitted values of m_H lower than (see Fig. 1) the direct experimental lower limit of 114.4 GeV at 95% CL. Replacing the fitted values of m_H by this lower limit, to give the largest possible CL consistent with all experimental data, will result in lower CLs than those quoted for the fits.

Finally are shown, in Fig. 8, the author's personal choice of the three most pertinent $\overline{\mathrm{CL}}$ curves among the 18 different ones previously presented in this paper. The 'A_l(lept) and m_W only' curve (dotted) gives the most reliable estimate of m_H. The value of $\overline{\mathrm{CL}}$ of $\simeq 0.2 - 0.3$ for values of m_H just above the direct lower limit of 114.4 GeV is quite acceptable. However for $m_H = 300$ GeV, $\overline{\mathrm{CL}}$ is $< 10^{-3}$ implying that, if the SM describes correctly the charged lepton sector and the Higgs boson exists, it must be very light indeed: ≤ 180 GeV if $\overline{\mathrm{CL}} \geq 0.05$. Higher values of m_H are favoured by the 'All data NuTeV out' curve (dashed). This is mainly due to the $A_l - A_b$ correlation following from (3,1) in the $A^{0,\mathrm{b}}_{\mathrm{FB}}$ measurement, as discussed above. The maximum value of $\overline{\mathrm{CL}}$ is $\simeq 0.05$ at about $m_H = 140$ GeV. Including the NuTeV measurement gives the 'All data NuTeV $\overline{s}_\nu$ meas.' curve (solid line) with a similar shape but lying roughly an order of magnitude lower. The maximum value of $\overline{\mathrm{CL}}$ is $\simeq 0.006$ at $m_H \simeq 180$ GeV. This accurately reflects the best possible level of agreement, with the SM prediction, of the entire electroweak data set. It is an order of magnitude, or more, lower than the CLs quoted by the EWWG and EWPDG groups for their global fits to similar data sets. It must be noted in closing, however, that the actual maximum value of $\overline{\mathrm{CL}}$ depends critically on the value of m_t. Varying the latter by plus or minus one standard deviation about the current experimental value changes $\overline{\mathrm{CL}}$ by roughly plus or minus one order of magnitude (see, for example, Fig. 4). Similar but smaller changes result from a variation of $\alpha(m_Z)$ (see Fig. 5). Improved measurements of these parameters are therefore essential for a more stringent test of the Higgs sector of the SM.

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